優質教育基金
Quality Education Fund

Faculty of Education
The University of Hong Kong
香港大學教育學院

Catering for Culturally and Linguistically Diverse Learners in Primary Mathematics Classrooms

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AF-ON
START/STOP
SET
LOCK

Service Highlights

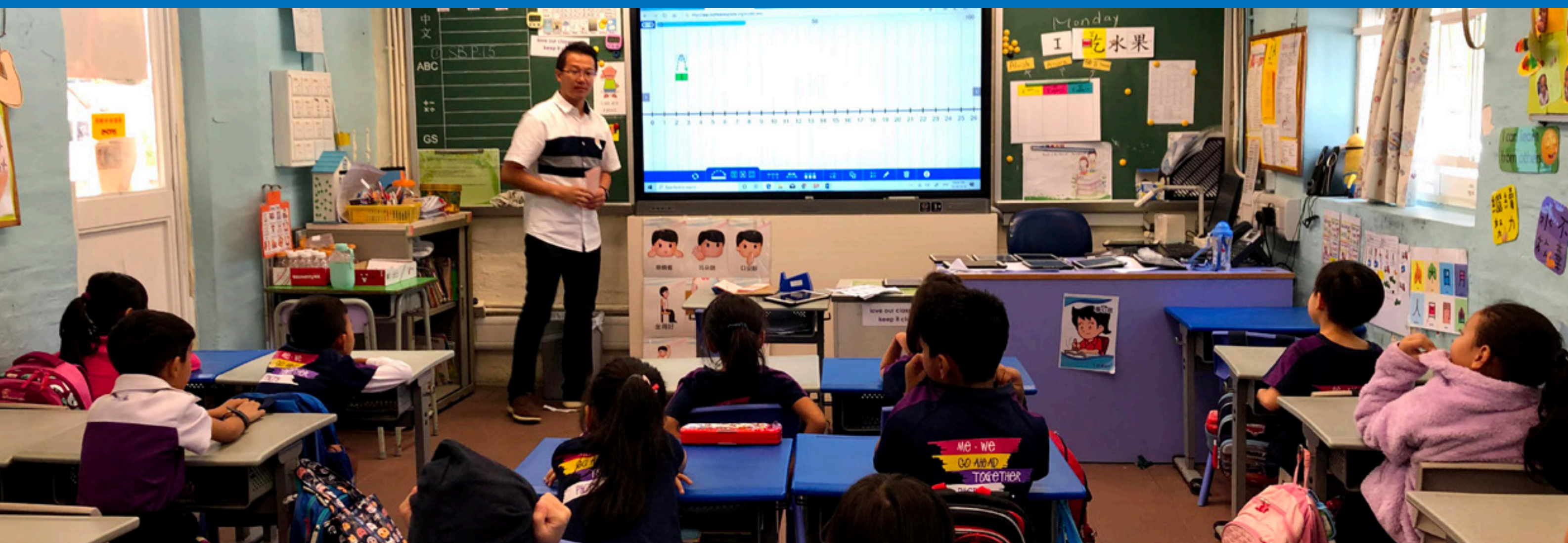
About the Project

The Project

- provides school-based support to mainstream primary schools with varying proportion of Culturally and Linguistically Diverse (CALD) learners, so as to enhance the mathematics learning of non-Chinese speaking (NCS) students;
- supports teachers' knowledge development and increasingly sophisticated instructional practices specific to CALD learners for long-term sustainability; and
- conducts research study and related work (including support services) on the teaching of mathematics in multicultural and multilingual classrooms.

Project Objectives

1. **Embrace cultural and linguistic diversity** (CALD) and enable NCS students to achieve their full potential in learning mathematics;
2. **Develop suitable school-based curriculum**, in alignment with the revised 2017 mathematics curriculum, to cater for CALD of school population of varying proportion of ethnic minority students amongst local Chinese students;
3. **Develop effective and diversified pedagogical strategies** (particularly those with the five strategic focuses mentioned in this booklet) to cater for the diverse learning needs of NCS students in classrooms of increasing CALD; and
4. **Design effective learning, teaching and assessment (LTA) materials** that facilitate NCS students' learning in mathematics, with more emphasis on their mathematical development and language acquisition.



About the People

Network Coordinator (Project Leader)
Mr WONG Ka Lok

Organisation-in-charge
Dr MOK Ah Chee Ida

Other Core University Members
Dr KI Wing Wah
Dr LEE Man Sang Arthur
Prof LEUNG K S Frederick

Project Manager
Ms CHOW Wing Sze Samantha (Jun 2021 – Aug 2021)
Ms LO Siu Yu Agnes (Sep 2020 – May 2021)

Curriculum Development Officers
Dr CHENG Wing Kin
Dr SUM Sze Wan Emily
Mr TAM Chi Leung Samuel
Ms WANG Dichen Daisy

Senior Research Assistant
Ms YIP Yu Ting Antonia

Experience and Achievements

2015-2017

USP Programme: Supporting the Learning and Teaching of Mathematics for NCS Students in Secondary Schools

- Explored the learning characteristics of NCS students
- Developed an understanding of the challenges/difficulties NCS students faced

2017-2019

USP Programme: Supporting the Learning and Teaching of Mathematics for NCS Students in Primary Schools

- Tried out a few pedagogical strategies to cater for NCS students in Numbers dimension
- Obtained preliminary research findings on students' values in mathematics learning

2019-2020

QTN-T Project: Supporting the Learning and Teaching of Mathematics for NCS Students in Primary Schools

- Developed the five pedagogical themes to enhance teachers' knowledge and build their teaching capacity through different strategies/tools in ALL dimensions
- Conducted research in culturally responsive mathematics education

2020-2021

QTN-T Project: Catering for Culturally and Linguistically Diverse Learners in Primary Mathematics Classrooms

USP: University-School Support Programmes
QTN-T: Quality Education Fund Thematic Networks – Tertiary Institutes

Modes of Delivery

Onsite School-based Support	2015-2017 (Secondary)	2017-2019 (Primary)	2019-2020 (Primary)	2020-2021 (Primary)
Primary / secondary schools	14	15	16	15
CLD classes	38	82	59	62
Students	759	1661	1066	1364
Teachers	46	100	90	73
Meetings with students	19	26	8	10
Collaborative Lesson Planning (CLP) meetings, Lesson Observation (LO), post-lesson evaluation meetings	260	233	201	278

Professional Learning for Teachers	2015-2017 (Secondary)	2017-2019 (Primary)	2019-2020 (Primary)	2020-2021 (Primary)
Orientation sessions	1	1	1	1
Thematic workshops	–	2	2	2
Joint-school workshops	14	3	2	2
Territory-wide disseminations	1	1	1	1
Mini online workshops	–	–	7	–
Maths book club meetings	–	–	2	–
Joint-school CLP meetings and LO	–	–	–	3

CRMT Framework

Culturally Responsive Mathematics Teaching (CRMT) is "using the cultural characteristics, experiences, and perspectives of ethnically diverse students as conduits for teaching them more effectively" (Gay, 2002, p. 106). The pedagogical framework recognises and affirms the diverse cultural backgrounds and the wealth of knowledge and experiences students bring to the classroom; it addresses the importance of including students' cultural references in all aspects of learning.

CRMT involves "a set of specific pedagogical knowledge, dispositions, and practices that privilege mathematical thinking, cultural and linguistic *Funds of Knowledge* (FoK), and issues of power and social justice in mathematics education" (Aguirre & del Rosario Zavala, 2013, p. 167).

The central role of a culturally and linguistically responsive teacher is to provide a learning environment in which students feel valued for who they are, and for their ways of engaging in mathematical thinking and reasoning for deep and meaningful learning.

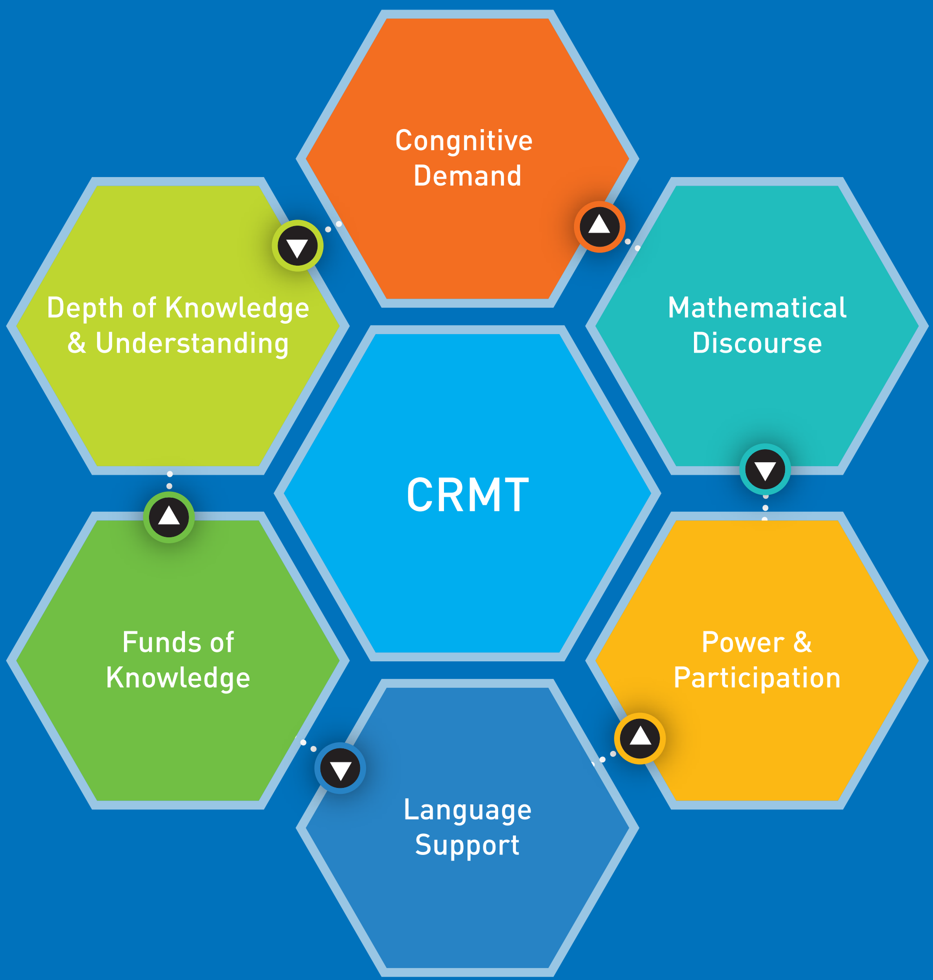


Figure 1: Conceptual Framework

CRMT Unit Planning / Lesson Observation & Analysis TOOL

(Adapted from Aguirne & del Rosario Zavala, 2013)

Cognitive Demand	• How does my lesson enable students to explore and analyse mathematical concept(s), procedure(s), and reasoning strategies? • Does it utilise multiple representations, and demand explanation or justification?
Depth of Knowledge & Understanding	• How does my lesson make student thinking / understanding more visible and deeper?
Mathematical Discourse	• How does my lesson create opportunities to discuss mathematics in meaningful and rigorous ways? – debate mathematical ideas / solution strategies; – use mathematical terminology, develop explanations; and – communicate reasoning, and make generalisations.
Language Support	• How does my lesson provide academic language support for Chinese / English language learners?
Power & Participation	• How does my lesson distribute mathematical knowledge authority, value students' mathematical contributions, and address status differences among students?
[Culture] Funds of Knowledge	• How does my lesson help students connect mathematics with relevant / authentic situations in their lives and communities?

References

Aguirre, J. M. & del Rosario Zavala, M. (2013). Making culturally responsive mathematics teaching explicit: A lesson analysis tool. *Pedagogies: An International Journal, 8*(2), 163-190.

Gay, G. (2002). Preparing for culturally responsive teaching. *Journal of Teacher Education, 53*(2), 106-116.

Romanisation of Chinese Characters

Learning Mathematical Vocabulary in Chinese

There is a lot of language learning going on in a mathematics class. The use of precise language of mathematics can help students effectively convey what they are thinking and doing when referring to mathematics. There are also many uses of mathematics vocabulary in students' general conversations, such as shopping and telling time. However, non-Chinese speaking (NCS) students often have difficulty in learning Chinese, which results in a delay in their mathematics vocabulary development. Some students show a lack of confidence to talk with their teachers and classmates, fighting the fear of being involved in any mathematics activities. Reading Chinese words and sentences is another problem that NCS students face when learning mathematics. With a limited Chinese vocabulary bank, students cannot always understand the questions and instructions in their learning materials. As most NCS students' parents do not speak Chinese either, students cannot seek help from their parents to read the texts in the assignments after school. As such, vocabulary learning support for NCS students is essential in mathematics education.

Chinese is not an easy language to learn. Compared to English and other European languages, there are no letters to communicate sounds in Chinese. Two Chinese characters look similar to each other, but their sounds could be very different, for example, "大" and "太", "火" and "炎". It makes learning Chinese daunting for many non-Chinese speakers.

Romanisation is a system of rendering a non-alphabetic language, such as Chinese, into alphabetic characters imitating the language pronunciation. It provides a useful way for foreigners who cannot recognise Chinese script to read and recognise Chinese instantly. Actually, romanisation is nothing new to anyone who visits Hong Kong. It can be commonly found on name cards, street signs, and at railway stations, where the Chinese name of a person or a place has been romanised. Romanisation helps non-Chinese speakers in Hong Kong produce sounds more closely resembling the Chinese words to communicate better with local people. This idea can also be used in supporting NCS students' learning of mathematics. NCS students can readily read and start to recognise Chinese characters on their own when they gradually familiarise themselves with the convention of romanisation.

When introducing new vocabulary in mathematics classroom, teachers could show the Chinese words with the romanisation. It could help NCS students imitate the pronunciation easily, which would reduce their burden in memorising the sounds and focusing on learning the meaning of the vocabulary. A flashcard designed as Figure 2 allows students to learn the sound and the meaning of each character of the vocabulary. Pictures could make the meanings even more memorable. Even the youngest learners enjoy identifying and pointing to figures, and now they can do it in Chinese. It is an easy starting point to learn Chinese words in mathematics.

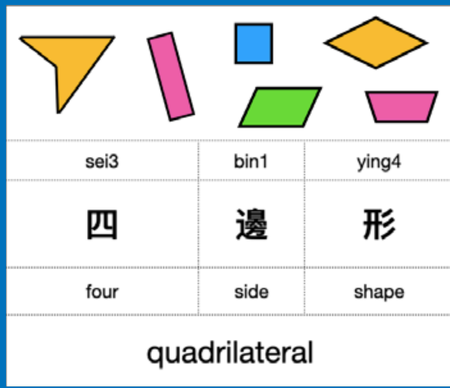


Figure 2

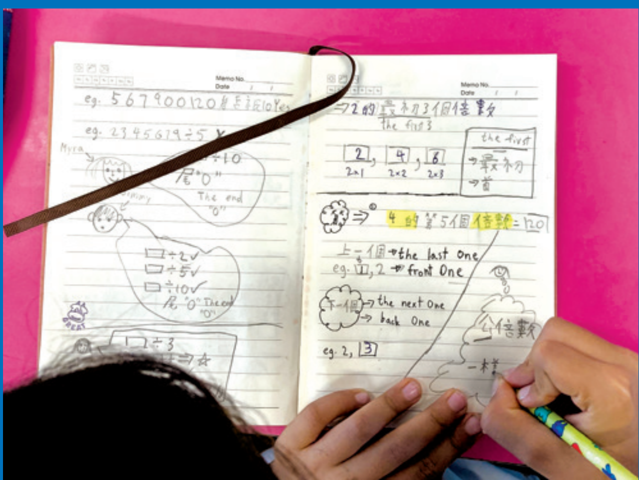


Figure 3

It is also a great idea to provide NCS students with a notebook to write down the mathematics vocabulary with the related romanisations and meanings. By referring to the notebook, students can review the pronunciation as well as the meaning of the vocabulary, and finally memorise everything. Teachers could let students present the meaning or the sound of a word in their own words, pictures, and methods of explaining that information (see Figure 3). It enables students to retrieve in their own ways what they have learnt in class and promotes studying through active recall.

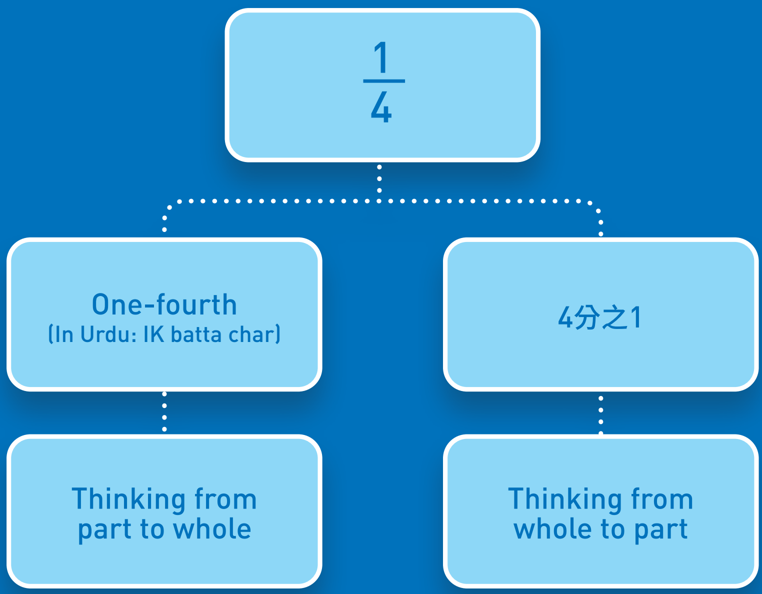
Language of Fractions

Fraction	English	Chinese	Korean	Japanese	Urdu	Nepali	Hindi	Filipino	Indonesian
$\frac{1}{2}$	One-half	二分之一	이분의 일	半分	آدھا	आधा	आधा	Kalahati	Setengah
$\frac{1}{3}$	One-third	三分之一	삼분의 일	三分の一	ایک تہائی	एक तिहाइ	एक तिहाइ	Isang-katlo	Sepertiga
$\frac{1}{4}$	Quarter/ One-fourth	四分之一	사분의 일	四分の一	ایک چوتھائی	एक चौथाई	एक चौथाई	Isang ikaapat	Seperempat
$\frac{2}{5}$	Two-fifths	五分之二	오분의 이	五分の二	دو پانچویں	पाँच भागको दुई भाग	दो पाँचना	Dalawang ikalimang bahagi	Dua per lima
$\frac{3}{2}$	Three- halves	二分之三	이분의 삼	二分の三	ایک اور آدھا	डेढ	तीन दुहाई	Tatlong kalahating bahagi	Tiga per dua
$4\frac{1}{4}$	Four and a quarter	四又四分之一	사와 사분의 일	四と四分の一	چار اور ایک چوتھائی	चार चौथाई	चार चौथाई	Apat at isang ikaapat	Empat seperempat

Reading Fractions in Two Languages / Directions

Mathematical concepts can be conceptualised in different languages (Barton, 2008) (see linguistic relativity hypothesis for more details). Leung (2017) points out the general differences between eastern analytic conceptualisation and western synthetic conceptualisation, with which the language of fractions seems to resonate. The part-whole concept is a good example of how different languages can provide different conceptualisations (Bartolini Bussi et al., 2014), and such differences concern not only the reading order, but also the articulation of the relationship between part and whole.

In Cantonese, the denominator of a fraction is read first and the numerator afterwards. The Chinese word 分 signifies partition(s) or the act of partitioning. For example, consider the fraction $\frac{1}{4}$. The English expression "one-fourth" literally means "one of the four partitions". But the Chinese expression " 四分之一 " literally means "four partitions, and take one of them" (4 等份，佔其中的 1 份). By expressing the denominator first, the act of partitioning is emphasised more strongly than in English. The order of articulation corresponds to the analytical way of thinking from whole to part and the part-whole relationship articulated is an integral part of the linguistic expression.



In English, the counting number (numerator) signifies "how many" while the ordinal number (denominator) signifies what is being counted. For example, taking "fourth" as a unit, we count "one-fourth"($\frac{1}{4}$), "two-fourths" ($\frac{2}{4}$), and "three-fourths" ($\frac{3}{4}$). That is, we count the unit fractions as entities. According to Watanabe (2002), "one of the strengths of English terminology seems to be the connections to the more familiar counting world. The challenge is to help children develop understanding of the fractional units with which they are counting"(p. 461). In other words, teachers using English as medium of instruction should support the synthetic way of thinking from part to whole; thus students may understand better what is being counted, i.e. the unit fraction.

The different language-related nuances of the conceptualisations call to investigate multilingual students in developing fractional concepts in and across languages. It may go beyond our project at this stage. Yet, understanding different ways to describe fractional quantities in different languages helps us (as well as teachers) better recognise the strengths and weaknesses of our ways of representing fraction concepts in words. Hence, it allows us to design instruction that best supports and caters for language diversity. Our teaching interventions and design aim at fostering students' fraction concepts with emphasis on language and representations to construct meanings.

References

Bartolini Bussi, M. G., Baccaglini-Frank, A., & Ramploud, A. (2014). Intercultural dialogue and the history and geography of thought. *For the Learning of Mathematics, 34*(1), 31-33.

Barton, B. (2008). *The language of mathematics: Telling mathematical tales*. Springer US.

Leung, F. K. S. (2017). Making sense of mathematics achievement in East Asia: Does culture really matter? In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education* (pp. 201-218). Cham: Springer.

Watanabe, T. (2002). Representations in teaching and learning fractions. *Teaching Children Mathematics, 8*(8), 457-463.

Activity First

Games and Activities

Benefits of Using Games in Learning Mathematics

There are many benefits of using games in and beyond mathematics lessons. Davies (1995) suggests that games motivate students to learn by providing students with meaningful situations, and develop their independence in learning. Orim and Ekwueme (2011) further point out the advantages of playing mathematical games at home that games allow parents and children to interact, communicate and discuss with each other, and provide opportunities for them to explore mathematical ideas together, which is normally only available inside the classroom.

To make the most of the experience, it is important that the mathematical activities can bring mathematics learning from school to home. In this way, students can consolidate what they learn from school and extend their learning at home. The activities should be easy to understand and enjoyable, too. It would be best if both parents and children can have fun and relax while engaging in the activities. For example, an activity that allows children to participate actively and be in control, or an activity that enables equal competition or cooperation between children and their parents or siblings would be much more enjoyable for all players.

The following parts suggest some family games that children and parents can play together, and how parents can make changes to the games so as to help develop children's mathematics skills or knowledge. The game "Snakes and Ladders" is used as an example.

Introduction

"Snakes and Ladders" is an Ancient Indian Board game called Moksha Patam. There are 100 grid squares on a board. The traditional "Snakes and Ladders" linked with Hindu Philosophy, Karma and Kama (meaning destiny and desire). See Figure 4. Images of different virtues are placed in the grids and there are snakes and ladders connecting different grids. This game was first introduced to Victorian England in 1892. It became popular to children and it was used as a tool in teaching good deeds versus bad. Since then, numerous versions of "Snakes and Ladders" have been created with different needs such as learning counting and having fun. Figure 5 shows a basic "Snakes and Ladders" board that can help students learn the counting of numbers from 1 to 100.

Figure 4: Gyan chaupar in National Museum, New Delhi

Activity 1 — Create Your Own "Snakes and Ladders" Board

Appendix 1: Basic 'Snakes and Ladders' board

100	99	98	97	96	95	94	93	92	91
81	82	83	84	85	86	87	88	89	90
80	79	78	77	76	75	74	73	72	71
61	62	63	64	65	66	67	68	69	70
60	59	58	57	56	55	54	53	52	51
41	42	43	44	45	46	47	48	49	50
40	39	38	37	36	35	34	33	32	31
21	22	23	24	25	26	27	28	29	30
20	19	18	17	16	15	14	13	12	11
1	2	3	4	5	6	7	8	9	10
Start									

Figure 5
Basic "Snakes and Ladders" Board

Game 1 – Counting with the Board

Download full instructions and game cards from here

Game 1a

Targets: Children aged 3 to 7 (Parents and siblings are helping)

Materials: Self-made "Snakes and Ladders" board, game cards

Parents count with kids. For example, "Let's count from the beginning." Parents and kids count and read out the numbers loudly. Parents set range of numbers to be counted. For example, "Please count from 24 to 41." Then kids start counting from 24 and read out the numbers loudly.

Game 1b

Targets: Children aged 3 to 7

Number of players: 2 to 4

Materials: Self-made "Snakes and Ladders" board, game cards

Parents or siblings cut out the game cards.

Each player draws a card and counts according to the requirements shown on the card. The player reads out the numbers loudly and other players check the numbers.

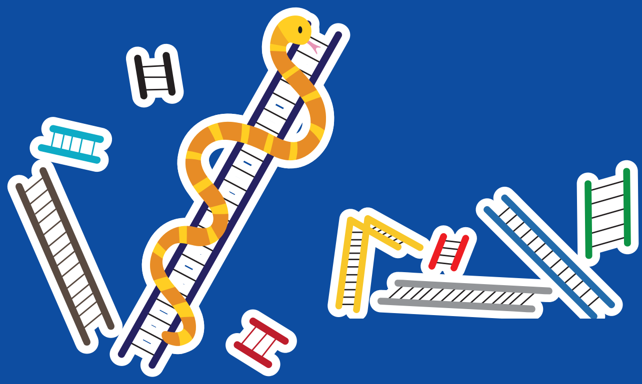
What is learnt?

In Game 1a and Game 1b, children learn to count, count backwards, count by 2, count by 10, count backwards by 2, and count backwards by 10. It develops kids' fluency in these counting skills, number sense and place value concepts.

Game 2 – Snakes and Ladders

Targets: Children aged 3 to 7

Number of players: 2 to 4



Game 3 – Snakes and Ladders (with Addition)

Targets: Children aged 6 to 8

Number of players: 2 to 4

Game 4 – Snakes and Ladders (with Multiplication)

Targets: Children aged 6 to 8

Number of players: 2 to 4

Download full
instructions from here

Summary

The above are some suggestions of mathematics learning activities that students can play with their family members. They enhance parent-children relationships and help build a happy family environment. Most importantly, children are learning mathematics in a fun way.

References

Davies, B. (1995). The role of games in mathematics. *Square One*, *5*(2), 7-17.

Orim, R. E., & Ekwueme, C. O. (2011). The roles of games in teaching and learning of mathematics in junior secondary schools. *Global Journal of Educational Research*, *10*(2), 121-124.

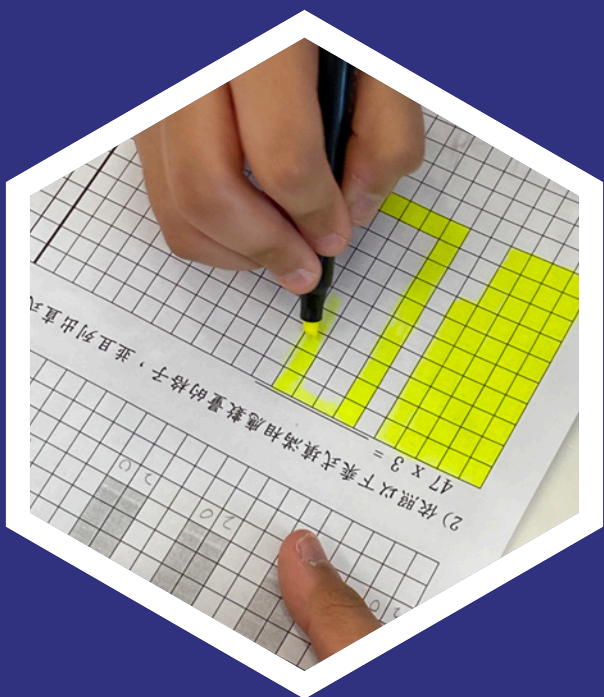
Beyond Algorithm

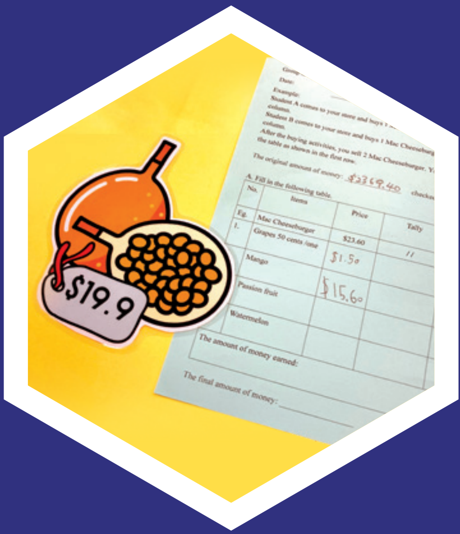
Students gain confidence in their ability to do mathematics when they use strategies that they understand.

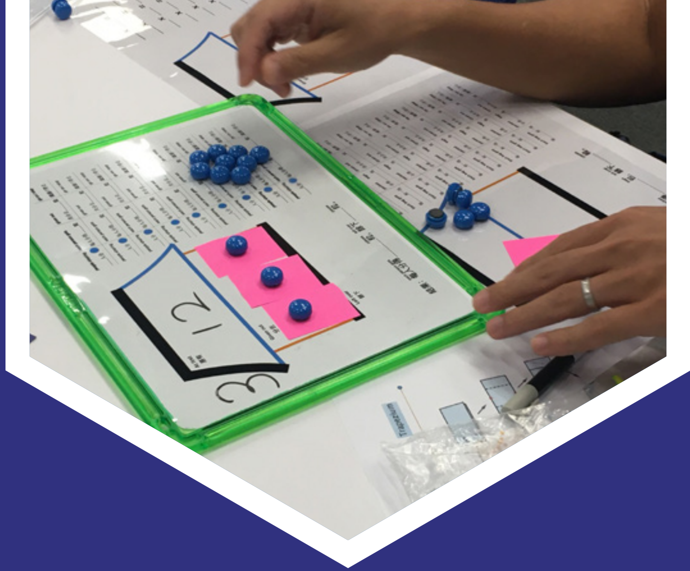
An *algorithm* is a step-by-step procedure designed to solve a problem. The most familiar algorithms in elementary mathematics are rules for adding, subtracting, multiplying, and dividing, but there are many other algorithms in mathematics. These algorithms are very powerful and have intrinsic value in their own right, rather than being a means to an end. However, the fact is that deeper mathematical understanding of algorithms is often lost in school mathematics.

The concept of the base-ten numeration system is fundamental to other mathematical concepts such as decimal numbers and exponents. Without a thorough understanding of place value, students can hardly perform standard procedures of addition, subtraction, multiplication, and division of multi-digit numbers, not to mention appreciating their elegance, nor being capable of scrutinising the validity of various deviants that may pop up during the learning process. Apart from the base-ten numeration system widely employed in daily life, place value extends its impact to decimals and the sexagesimal (base-60) numeration system adopted in calculating and recording time.

How can we guide students to understand place value? Freudenthal (1983) maintains that mathematics learning should start from "those phenomena that beg to be organised and from that starting point teaching the learner to manipulate these means of organising" (p. 32). To begin with, students need to closely experience and examine the idea and phenomenon of putting things in groups of a specific number.







Give students two piles of beads, one pile of them has been grouped in tens, and the other pile has not. Let students observe the process of getting 25 beads from each pile. After the observation, teachers or parents can lead students to compare the efficiency of them. With a discussion on the difference between getting beads from the two piles, students conclude that "grouped in tens" can improve the efficiency of getting many objects. It motivates them to group all the beads in tens.

Mathematics contents behind the above activities are:

1. Ten ones make a ten;

2. A ten is comprised of ten ones;

3. In a system where the counters are grouped in tens, there will only be two units: tens and ones. The number of ungrouped ones should be less than ten. Otherwise, it can make a new ten.

The next activity is to consolidate their understanding of numbers by tens and ones. Students are given numbers less than 100, and they need to use the bead strings (tens) they made and the remaining separate beads (units) to make the amount corresponding to the numbers given. This activity is important because it can help them translate any two-digit numbers into manipulatives, by which they can explore arithmetic in concrete terms.

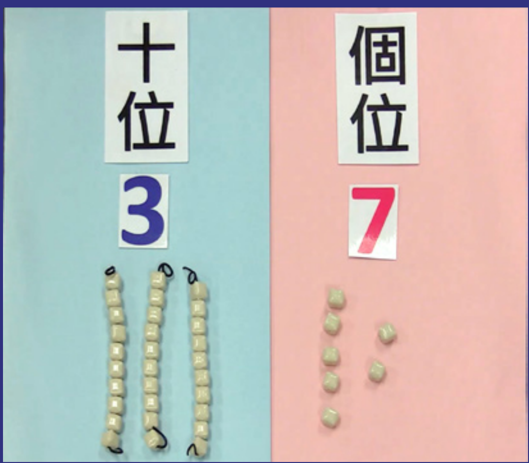
After students become accustomed to thinking in terms of tens and ones, the same strategy is used to motivate students to create a larger unit --- making ten tens into one hundred. A key indicator of the extent to which students have developed a sound basis for place value is the extent to which they can efficiently count large collections and confidently name, compare and sequence multi-digit numbers, and count forwards and backwards with them.

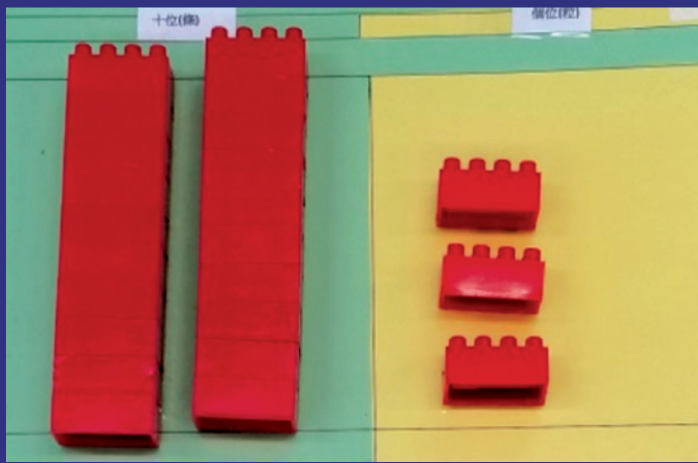
The following parts suggest some activities that students can develop the concepts of place value with family-accessible objects:

- Estimate the total

  Fill a clear jar with 50-100 items, such as beans, paper clips, rubber bands. Allow students to write down their estimates of the number of items in the jar. Then, let students group the items ten at a time and tell how many items there are altogether.

- Learn tens and ones by bead strings or LEGO® blocks





- Place Value War

  Play this game with Uno cards or other cards with the numbers 0-9. Each player gets three cards to build the highest possible three-digit number.

Reference

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Dordrecht Boston: Reidel Pub. Co.

Culture Matters

Teachers can bring in students' cultural funds of knowledge (FoK) into their mathematics teaching, and make explicit connections from what students already know at home to what is being taught in class. Culturally familiar materials, such as recipes from around the world, are good resources to support mathematics learning. Students mathematise their world through cultural food. They can explore different mathematical concepts such as numbers, fractions, counting, measurement and shapes, through well-designed activities based on cookbooks and recipes. Besides, linking new mathematical language to students' own personal experience improves their mathematics understanding, and word problem comprehension.

Example

Tandoori Roast Chicken

A dish originated in the Punjab, and is very popular among Punjabi, Pakistani and Indian.

Ingredients

1.8kg chicken

2 onions, thickly sliced

1 lemon, halved

thumb-sized piece ginger, peeled and thickly sliced

400g can coconut milk

small bunch coriander, roughly chopped

For the marinade

150ml pot natural yogurt

1 tbsp tomato purée

juice 1 lemon

1 tsp each hot chilli powder, turmeric, ground coriander, ground cumin, garam masala and ground cinnamon

6 garlic cloves, whizzed to a paste with the ginger

½ finger-length piece ginger, whizzed to a paste with the garlic

few drops red food colouring (optional)

Method

STEP 1

Mix the marinade ingredients with 2 tsp salt and 1 tsp black pepper. Slash the legs of the chicken a few times, then rub the marinade all over, including under the skin of the breast. Marinate in the fridge for up to 24 hrs.

STEP 2

Heat oven to 200C/180C fan/gas 6. Put the onions, lemon halves and ginger in a roasting tin. Sit the chicken on top and roast for 1½ hrs or until the thigh juices run clear when tested with a skewer.

STEP 3

When the chicken is done, lift out of the tin, sit in a new dish, cover loosely with foil and leave to rest. Fish out the ginger from the tin and discard. Scrape out the roasted middles from the lemons into a food processor, add the onions and any pan juices, and whizz to a purée. Scrape the purée back into the tin and sit on the hob. Stir in the coconut milk and bubble gently, scraping up any chicken bits that have stuck. You can add a splash of water if the sauce is too thick. Stir in the coriander and serve with the chicken.

Teachers can talk about numbers and quantities, introduce new words and practise pronunciations, such as fraction, one-half, numerator, denominator, equal, unequal and whole. Then ask students to estimate and compare the size of different ingredients. They can use non-standard units (thumb-size, small bunch), and use measuring tools such as measuring spoons (tsp, tbsp) and cups (g, ml) to understand standard units. Students can also learn about temperature and time duration (cooking time). Teachers can introduce the concept of ratio and proportional reasoning by increasing and decreasing the number of servings of the recipe. For instance, if the number of servings of chicken is halved, then the amount of ginger needed also needs to be halved, to only a quarter. Students can do the computation by making sense of the fractional quantities or by modelling the context. Using the recipe context to teach multiplication and division of fractions and mixed numbers helps students make sense of the problem solutions with concrete referents, and avoid relying too much on the standard algorithms. Incorporating students' cultural FoK into mathematics teaching, which usually goes ignored and unrepresented in textbooks and other materials, shows that their lives, traditions and cultures matter.

Depth with Fluency

Use grids to design activities to cater for students' diversity in spatial sense.

In learning primary geometry, students may memorise formulas and rules without depth of understanding. They may mix up base, length, width, height, etc. The $\frac{1}{2}$ in the area formulas makes little sense to them. It is not uncommon for students to possess the misconception that "larger in area means longer in perimeter", even if they can distinguish area from perimeter. Students start calculating areas by formulas before really knowing how large 1 cm^2 is.

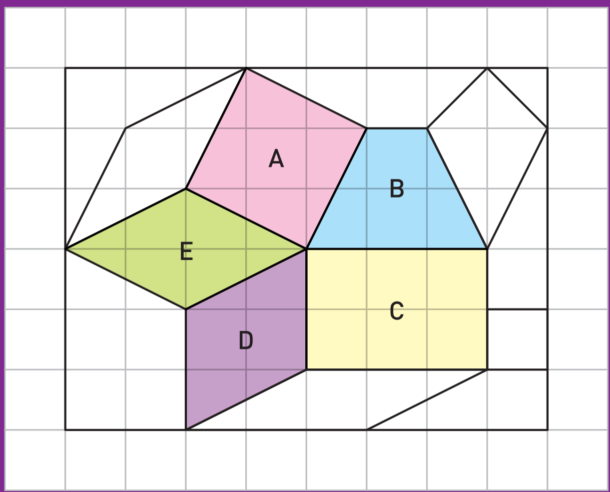
Students identify types of figures in lower primary by appearance. When inclusive relationships among quadrilaterals are introduced, they feel confused. For example, how can rectangles be parallelograms? Students can be given more opportunities to work on different shapes at the same time so that they can distinguish their features and relate them to different formulas.

The following activities use 1cm x 1cm grids (or 5cm x 5cm) so that every figure can be measured, investigated and students can grasp the concept of area via exploration. With their experience of measuring and drawing, they may discover formulas or compare formulas they learnt about perimeter and area.

Exercise books with grids or dots could be bought from stores, provided by teachers, or downloaded from the internet.

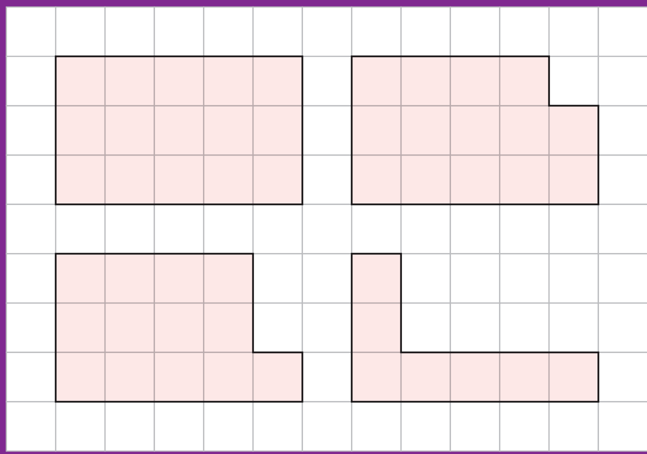
1. Polygon Wall

- What kinds of figures are A, B, C, D and E? Try to measure and explain your answers.
- What other figures can be found? Explain your answer.
- On grid paper, make your own polygon wall.

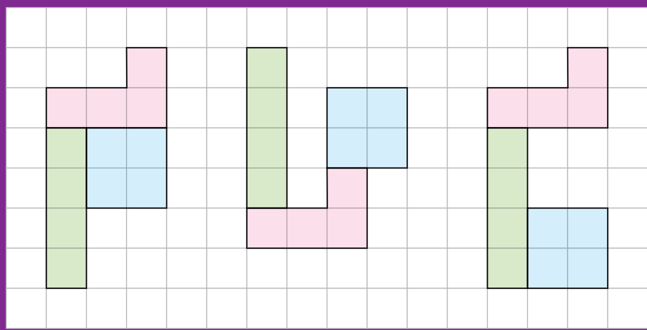


2. Perimeter and area are not related

- Find the area and perimeter of the figures on the right. What do you observe?



- Find the area and perimeter of the figures on the right. What do you observe?



- On the grid paper provided, draw two rectangles. One is larger in area but has a shorter perimeter than the other.

  Online practice: *https://www.geogebra.org/m/vfwawxqp*

3. Making Polygon Game (2 Players)

i. On grid paper, Player One draws a figure with an area not bigger than 12 cm^2.

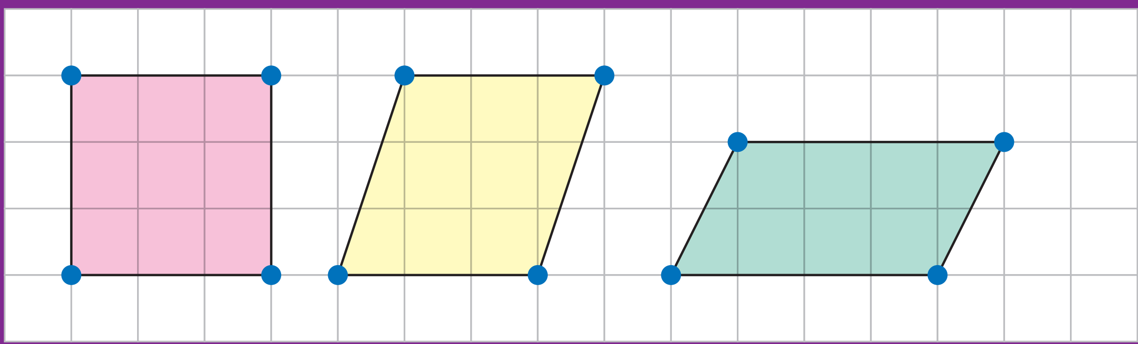
ii. The two players play rock-paper-scissors to determine what the next figure should be:

- If Player Two wins, he/she draws a bigger figure of the same type.
- If Player Two loses, he/she draws a smaller figure of the same type.
- If it is a draw, Player Two draws a figure with the same area but of a different type.

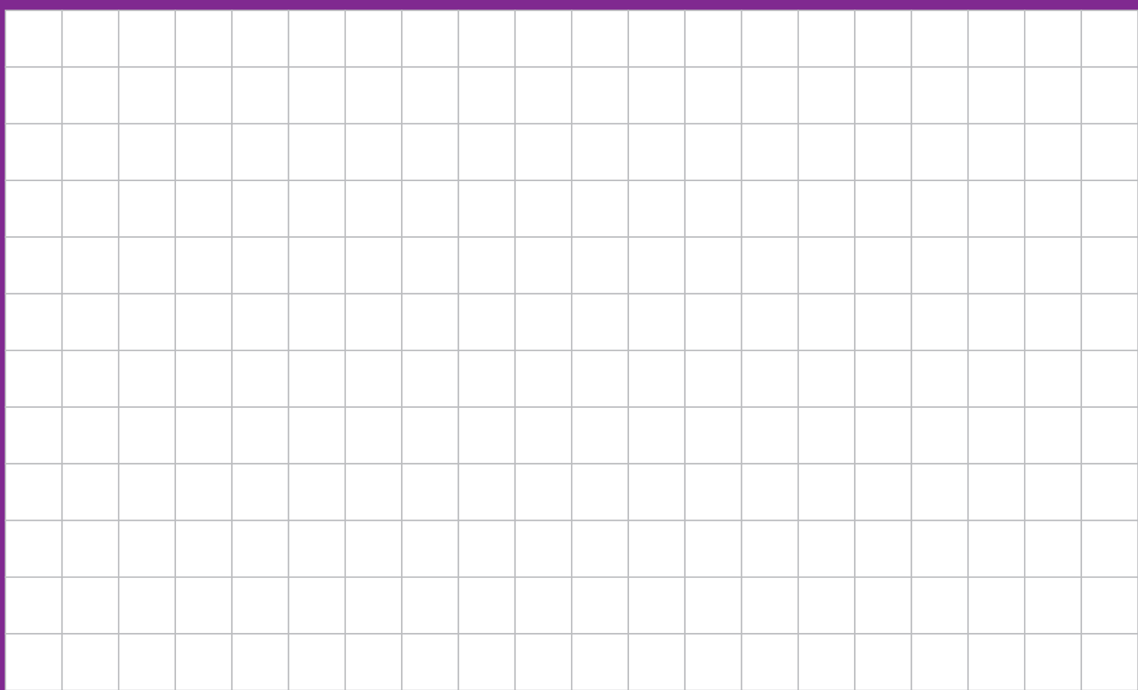
iii. The two players play rock-paper-scissors and take turns to draw the next figure.

iv. Lastly, the two players verify the correctness of each figure.

Example: Player One draws a square. The two players play rock-paper-scissors and it is a draw. So, Player Two draws a parallelogram with an equal area. They continue to play rock-paper-scissors and Player One loses, he/she draws a parallelogram with a smaller area.



Enjoy the game



Exercise Counts

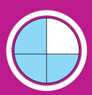
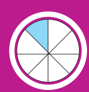
In the learning and teaching of mathematics, exercise is important because we expect students to practise certain skills. For the learning of primary mathematics, developing proficiency with various computational techniques is crucial to the success in progressing smoothly through the curriculum. Without disregarding the importance of getting the students familiar with basic routines of calculation and more proficient in handling typical problems, we can try different types of exercise which, if properly used, are likely to engage the students in more active thinking and reasoning while practising a particular skill.

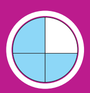
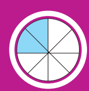
Example 1: Exercise on Division of Fractions

Set A is typically found in textbook exercise. The selection provides opportunities for the students to do the division of fractions when dealing with various numbers in the numerator and denominator. The purpose is clearly set on practising the division routine. **Set B** also allows the practice of the basic routine of dividing a fraction by a fraction; but the students will be exposed to a much more restricted choice of numerators and denominators.

Set A	Set B
$\frac{3}{4} \div \frac{1}{8}$	$\frac{3}{4} \div \frac{1}{8}$
$\frac{5}{6} \div \frac{2}{3}$	$\frac{3}{4} \div \frac{2}{8}$
$\frac{8}{9} \div \frac{4}{7}$	$\frac{3}{4} \div \frac{3}{8}$
$\frac{3}{10} \div \frac{2}{5}$	$\frac{3}{4} \div \frac{4}{8}$

For this reason, **Set B** may thus be considered as less effective in training the computational skills. However, at the beginning stage of getting the students more familiar with the division of fractions, Set B may create an opportunity for students to gain a better sense of the division – especially when a couple of questions can be asked upon the completion of the four divisions in **Set B**. For example,

If  (three quarters of a pie) is divided into pieces of  (one-eighth of a pie), how many pieces can you get?

If  (three quarters of a pie) is divided into pieces of  (two-eighths of a pie), how many pieces can you get?

Example 2: Exercise on Simple Addition

8 + 5 =	7 + 6 =	15 + 3 =
9 + 5 =	9 + 6 =	5 + 3 =
10 + 5 =	11 + 6 =	5 + 13 =
11 + 5 =	13 + 6 =	15 + 13 =

Similar to **Set B** in **Example 1**, the above series of exercise on addition do not seem to provide sufficient drilling opportunities with different numbers. However, upon completing the problems (column-wise), the answers will reveal a certain pattern which should trigger a mindful learner to think about the relationships underlying the numbers involved. If the teacher (or the parent) wants, an extra question can be asked of the student: *Can you create a set of addition problems similar to this one?*

Opportunities for Building Connections and Reasoning with Patterns

The examples here highlight the design of exercise which is not simply a bunch of unconnected practice items concerned only with the drilling of a particular skill. Attempts are being made to create opportunities for the students to reason and make connections between numbers and the respective calculations. Such calculations are practices. Moreover, when going through the calculations arranged in a particular way, the students may attend to patterns, structure or mathematical relationships, thus enjoying an opportunity to deepen conceptual understanding.

Example 3: Exercise on Subtraction

This is an exercise of subtraction with three-digit numbers. But, it also involves a practice with the concept of place value (in Step 2 below) as well as an opportunity for an interesting discovery.

1. Start with a three-digit number with all different digits.
2. Rearrange the digits to make the largest and smallest numbers possible with the three digits.
3. Subtract the smallest from the largest.
4. With the result of Step 3, repeat from Step 2 above.
5. Repeat until you notice something special.

Try a few other three-digit numbers, following the above steps.

If you are familiar with four-digit numbers, you can do the same with any four-digit number (with all four digits different). Do you arrive at similar observation as with three-digit numbers?

Example 4: Exercise with Numbers on a Calendar

Pick any month of the year from a calendar.

For example, October 2021 is chosen on the right.

OCTOBER 2021

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

Use a square (2 x 2) frame to group any 4 numbers on the page. Add the two numbers along each diagonal. What do you observe? Check if it is true for other groups of 4 numbers.

Try a bigger square (3 x 3) frame to group any 9 numbers. Do you arrive at a similar result? Go get your friends to try with other months.

Try another rectangular frame (whatever size that may fit onto the month) that groups a different number of numbers. Do you come up with similar observation?

Summary

With the exercises and tasks designed in a thoughtful way (in addition to mechanical drilling), students are working on many calculations which are not simply at the order of the teacher but naturally generated or partly by the students themselves according to certain mathematical principles and/or out of their curiosity about certain unexpected results or patterns.

Recommended Storybooks

Children's literatures that you can read in class, at home... and anywhere!

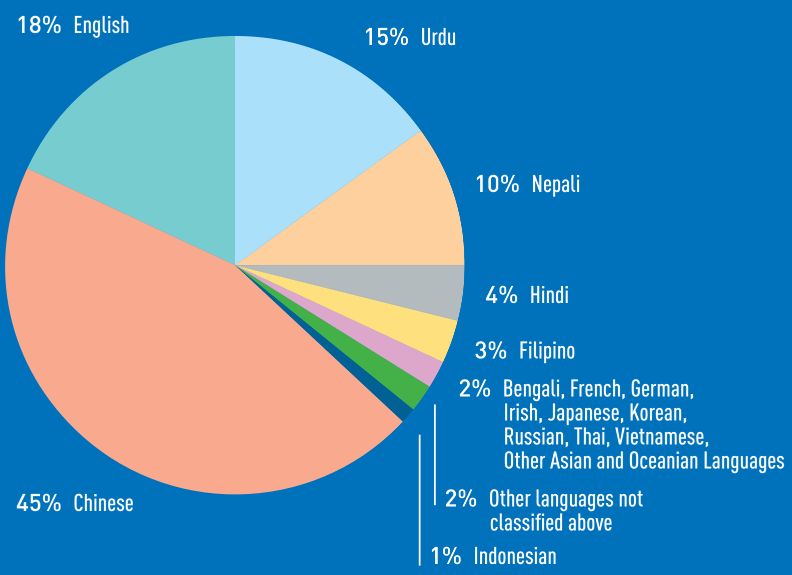
1. 美味的馬鈴薯濃湯. *陳忠勤 (譯)*
2. Spaghetti and Meatballs for All! *Marilyn Burns*
3. Have You Seen My Monsters? (你有看到我的怪獸嗎？). *Steve Light*
4. What's Your Angle, Pythagoras? *Julie Ellis*
5. The Greedy Triangle (貪心的三角形). *Marilyn Burns*
6. The Shopping Basket. *John Burningham*
7. Mr Owl's Bakery: Counting in Groups. *Ji-Hyeon Kim*
8. Fibonacci Zoo. *Tom Robinson*
9. Mystery Math - A First Book of Algebra. *David A Adler*
10. The Boy Who Loved Math - The Improbable Life of Paul Erdös (熱愛數學的男孩). *Deborah Heiligman*
11. Everyone can Learn Math. *Alice Aspinall*
12. The Girl With a Mind for Math: The Story of Raye Montague (有數學頭腦的女孩). *Julia Finley Mosca*
13. The Best Food in the Forest, Picture Graphs. *Mi-Ae Lee*
14. Wangari's Tree of Peace: A True Story from Africa. *Jeanette Winter*
15. Annika Riz, Math Whiz. *Claudia Mills*
16. Clean-Sweep Campers. *Lucille Recht Penner*
17. Fractions = Trouble! *Claudia Mills*
18. Jalapeno Bagels. *Natasha Wing*
19. Mummy Math: An Adventure in Geometry. *Cindy Neuschwander*
20. One Snowy Night. *Seon-hye Jang*
21. Twinderella: A Fractioned Fairy Tale. *Corey Rosen Schwartz*

Statistics of Students in 2020-21

Spoken Language at Home

(total number of students = 1364)

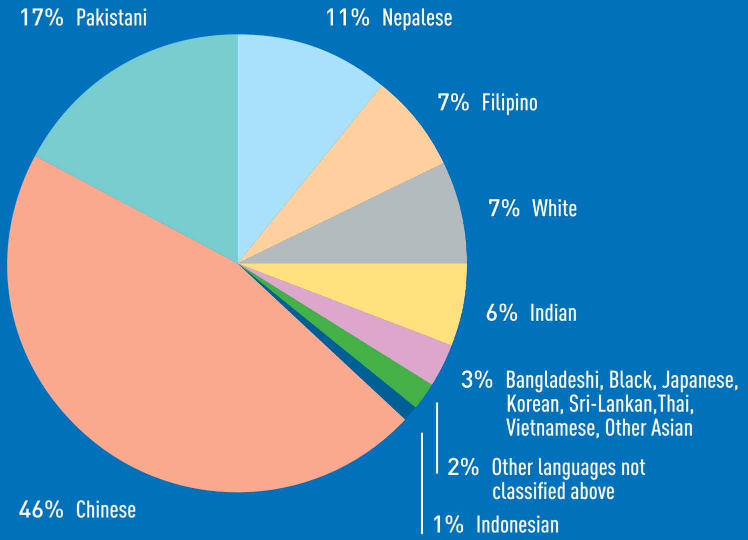
Chinese	45%
English	18%
Urdu	15%
Nepali	10%
Hindi	4%
Filipino	3%
Bengali, French, German, Irish, Japanese, Korean, Russian, Thai, Vietnamese, Other Asian and Oceanian Languages	2%
Other languages not classified above	2%
Indonesian	1%



Ethnicity

(total number of students = 1364)

Chinese	46%
Pakistani	17%
Nepalese	11%
Filipino	7%
White	7%
Indian	6%
Bangladeshi, Black, Japanese, Korean, Sri-Lankan, Thai, Vietnamese, Other Asian	3%
Other ethnicities not classified above	2%
Indonesian	1%



Acknowledgements

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1. Bui O Public School
2. Catholic Mission School
3. Chan's Creative School (Hong Kong Island)
4. CNEC Ta Tung School
5. Delia (Man Kiu) English Primary School
6. Hong Kong Christian Service Pui Oi School
7. Hong Kong Taoist Association Wun Tsuen School
8. Islamic Primary School
9. Kam Tin Mung Yeung Public School
10. Northern Lamma School
11. Pat Heung Central Primary School
12. Po Leung Kuk Gold & Silver Exchange Society Pershing Tsang School
13. Sir Ellis Kadoorie (Sookunpo) Primary School
14. St. Margaret's Co-educational English Secondary and Primary School
15. Tsuen Wan Trade Association Primary School

HKU NCS Math

3910 2412

cldmaths@hku.hk