Culturally Responsive Mathematics Teaching:

Children's Literature as a Pedagogical Tool to Develop Mathematical Concepts

by Emily S.W. Sum

We understand something if we see how it is related or connected to other things we know (Hiebert et al., 1997, p. 4).

The use of children's literature to develop mathematics concepts is nothing new, we have started using storybooks as a springboard for exploring mathematics ideas, especially in lower primary levels. It also provides opportunities for teachers to instil problem-solving skills at an early stage. Most mathematics teachers will agree that students, especially culturally and linguistically diverse (CLD) students, have difficulties in solving word problems. Many word problems in textbooks are unauthentic and irrelevant to the lives of CLD students. Students may not have the same background knowledge related to the word problem, nor the understanding of the language and the context of the problem in order to solve.

Mathematical ideas arise naturally from stories, in which students can locate the relevant quantities in the story and make sense of the quantities and their relationships. It is important for teachers to make explicit connections between the story and the mathematical concepts in the curriculum, also to promote classroom discussions in communicating mathematically. Storybooks, as a pedagogical tool, support teachers in integrating multiple resources into their mathematics lessons, and make teaching more culturally responsive. Culturally Responsive Mathematics Teaching (CRMT) emphasizes mathematical thinking, culture and language, as described in Aguirrea and del Rosario Zavalab (2013):

Culturally responsive mathematics teachers leverage mathematical learning by expanding children's mathematical thinking, building bridges between previous knowledge and new knowledge, supporting bilingualism and academic languagedevelopment, fostering connections with cultural funds of knowledge and experiences, andcultivating critical mathematical knowledge that enables students to analyse and address authentic problems (p. 168).

CONNECTING MATHEMATICS AND LITERATURE

The book ***100 hungry ants*** **(好餓好餓的螞蟻)** was previously selected to provide a context to explore the relationship between multiplication and division in Primary two (see xxx for details). One hundred hungry ants head toward a picnic to get food in different line formations. The story is rich with mathematical possibilities. It provides an authentic context for mathematical ideas that promotes reasoning and problem solving. Not only is the operation of multiplication inextricably connected with the operation of division, but the concept of multiplication also plays a role in many mathematical concepts in upper primary levels. These include factors and the highest common factor, factorization and prime factorization, multiples and the least common multiple, prime and composite numbers, and area. Reading the story while learning about topics such as multiplies, factors, primes and composites provides a context of imaginative representation in which students can model a creative experience and discuss their understanding of multiplicative situations, thereby making the mathematical content more comprehensible. The book was selected once again in the unit of Factors and Multiples, a total of … lessons in Primary four at a CMI school. Three teachers were involved in this case study, in which they attended to the mathematical needs of language learners through activating their prior knowledge and other visual and tactile resources.

FACTORS (因數) AND MULTIPLES (倍數)

Students construct their knowledge through social interaction that is mediated by language and culture (Cobb & Bowers, 1999). Language is a tool to connect mathematical ideas, thus we must place adequate focus on vocabulary development in different contexts, and through various ways to engage students in making sense of the world around them mathematically. As we can see later, through direct or indirect means, teachers introduced new vocabulary items by explaining what they mean explicitly with examples of its usage in the story. They deliberately used language strategies such as animations/illustrations, gesturing and other graphic organizers during whole class instruction (when small group discussion was not allowed due to social distancing in the pandemic). Students were asked to analyse the mathematical situations and solve problems in different contexts, which helped them make sense and connect mathematics with authentic situations in real lives.

MULTIPLES (倍數)

To begin the lesson, teachers showed the illustrations from the storybook (see Fig). They used the characters from the story and discussed the concept of multiplies and introduced the term "multiples (**倍數**)" based on students' prior knowledge on multiplication. For example, Ms. Choi initiated the discussion by saying "**出發前要點人數**", and asked students to complete the tables "**你能搵出以下的數目嗎?**" When finding the number of legs of 2 ants, one student said "six plus six" and another one said he used multiplication to find the number. Students worked with the function tables using their own strategies, and explored patterns/relationships among quantities that vary in relation to each other.

螞蟻（隻）	嘴（張）	眼睛（隻）	腿（條）
1	1	2	6
2	2	4	12
3	3	6	18
4	4	8	
5	5		
6			
10			
12			
50			
100			

To conclude the first lesson, Ms. Choi posed an interesting question "**我們可以找出一個數的最大倍數嗎? 為甚麼?**" This set the foundation for thinking multiplicatively throughout the unit.

FACTORS (因數)

In the second lesson, teachers read the story "one hundred ants were singing and marching in a row (**有100隻好餓好餓的螞蟻在排隊**)" and played with the plot to prompt mathematical ideas and evoke mathematical thinking. A hundred hungry ants marching in one row worried that food would be gone before they arrived.

Lining up is something students encounter in school and everyday life. The plot connects with students' life experiences and existing knowledge, and makes mathematics **relevant** and meaningful. A T-chart or T-table (see Fig) was used as a graphic organiser to visually represent the information, and to record the number of rows and the number of ants in each row. Students used different strategies to find the number of rows, including division.

Teachers posed other patterns/possibilities by asking students "what if" questions. For instance, Ms. Choi asked "What if 100 ants line up in 3 rows? Why not? **如果**100**隻螞蟻排成三行得唔得? 點解唔得?**" She pressed for a reason "**俾個理由我… 點解唔得?**" A student replied "**唔平均**". She used animation to illustrate the arrangement (see Fig). Some students said "**多咗隻**", she revoiced "**唔可以㗎會嗰交~ 要平均齊齊整整**" and concluded "**我哋冇辦法用三組合而成**". This example illustrates how the use of story can provide a context for the development of mathematical communication, as stated in Cummins (p. 68)

in context-embedded communication the participants can actively negotiate meaning (e.g. by providing feedback that the message has not be understood) and the language is upported by a wide range of meaning-ful interpresonal and situational cues.

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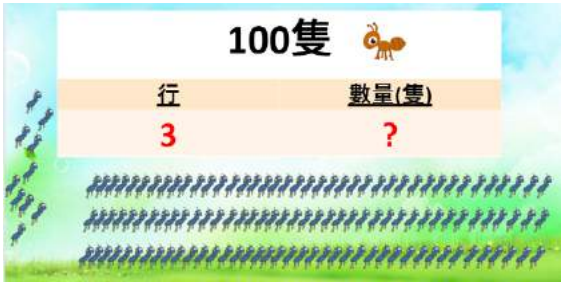
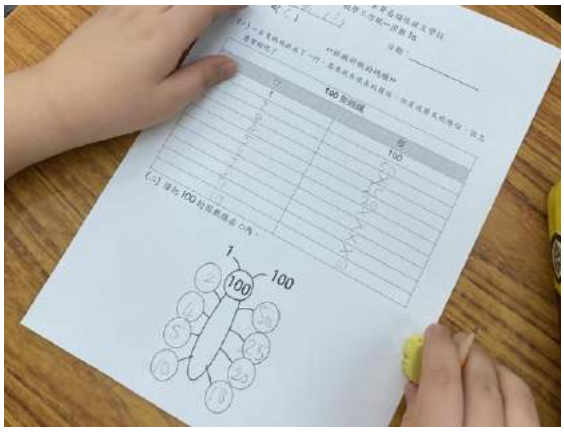




Ms. Choi continued with other numbers 4 to 9, and explored different line formations using division and array models. As the story concluded, students completed the table and explored the factors of 100.

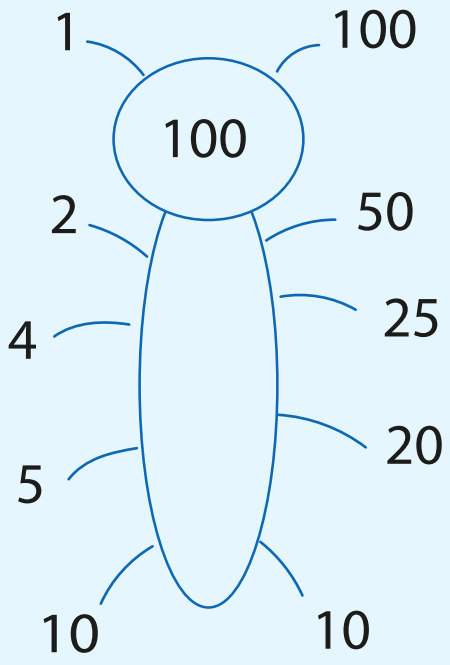
STUDENT ACTIVITY SHEET

《好餓好餓的螞蟻》

1. 一百隻螞蟻排成了一行，需要很長很長的隊伍，但是這麼長的隊伍，該怎麼變短呢？

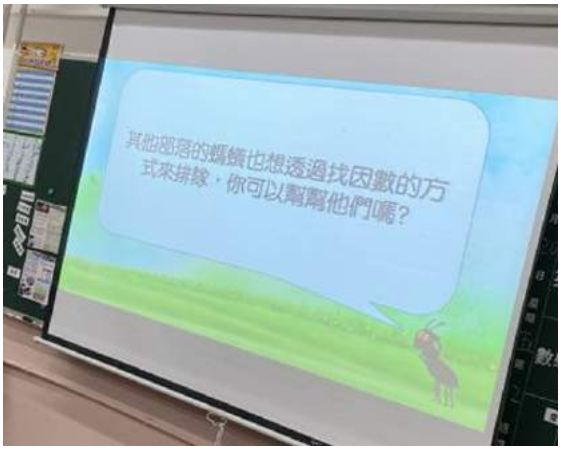
100隻螞蟻	
行	隻
1	100

2. 請把100的因數填在□內。



FINDING FACTORS

The story was retold with small changes. Factors of 4, 8, 9, 12, 15, 16, 28 and 20 become apparent as students explored the number of ants using array models and recorded on the tables (see Fig). The term, factor (**因數**), was introduced and defined through the task. Finding factors is an important skill, especially when we want a fraction to be expressed in the simplest form, that is, with both the numerator and denominator as counting numbers that have no common factors other than 1. Therefore, it is important to develop students' conceptual understanding as well as procedural skills.



(1) 如果現在只有 18 隻螞蟻，可以有甚麼排列方法？

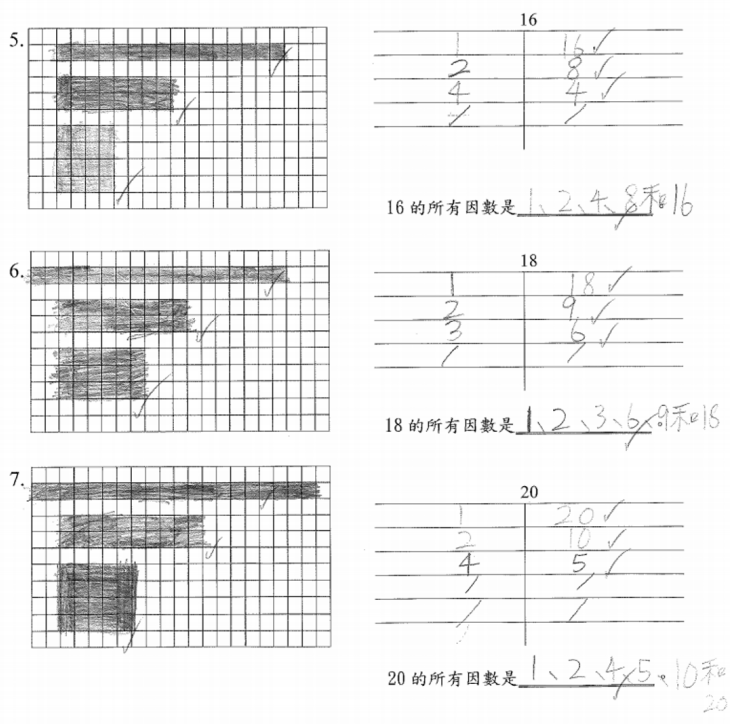
18 隻螞蟻

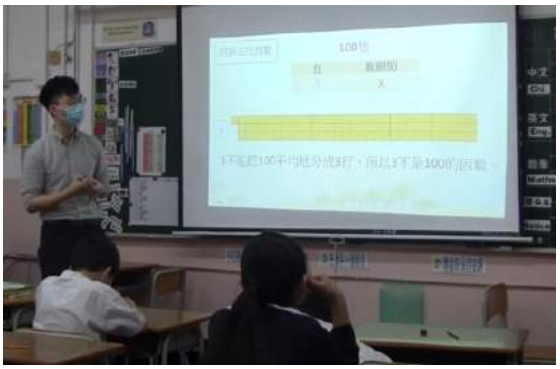
行	隻
1	18 ✓
2	9 ✓
3	6 ✓
4	×
5	×
6	3 ✓
7	×
8	×
9	2 ✓

18 的所有因數有: 1, 2, 3, 6, 9, 18 ✓

STUDENT WORK

利用方格圖找出以下各數的所有因數。





Teachers discussed the ideas using different mathematical representations such as tables and array models (using drawings and digital blocks on iPads). Multiple representations allow students to communicate their thinking and understanding while they develop command of the Chinese language and mathematics. For example, Mr. Fung used academic language "3**不能把**100**平均地分成**3**行, 所以**3**不是**100**的因數**" while also showing the array models to illustrate the vocabulary "**平均**". Through the systematic use of diagrams, illustrations and language, students begin to understand and employ problem solving strategies in real-world situations. This also improves student learning, retention and promotes a deeper understanding of the mathematical concepts.

COMMON FACTORS (公因數) AND COMMON MULTIPLES (公倍數)

Teachers modified the story plot and created new mathematical problems using an idea that the class had explored and studied to help make sense of common factors and highest common factor. There was a rich connection to students' life experiences, while promoting equity and fairness in the learning of mathematics. Teachers guided students to become critical thinkers and problem-solvers through the inquiry process.

COMMON FACTORS (公因數) AND THE HIGHEST COMMON FACTOR (最大公因數)

……在經歷上一次失敗後，螞蟻學會了用因數的方式排隊。可是……一天，螞蟻A隊和B隊同時出外找食物……你們12隻螞蟻，分作12行，路都被你們霸住了！要怎樣做才公平？

B隊: 8隻螞蟻	
行	隻
1	8
2	4
4	2
8	1

8 的因數有：1,2,4,8

A隊: 12隻螞蟻	
行	隻
1	12
2	6
3	4
4	3
12	1

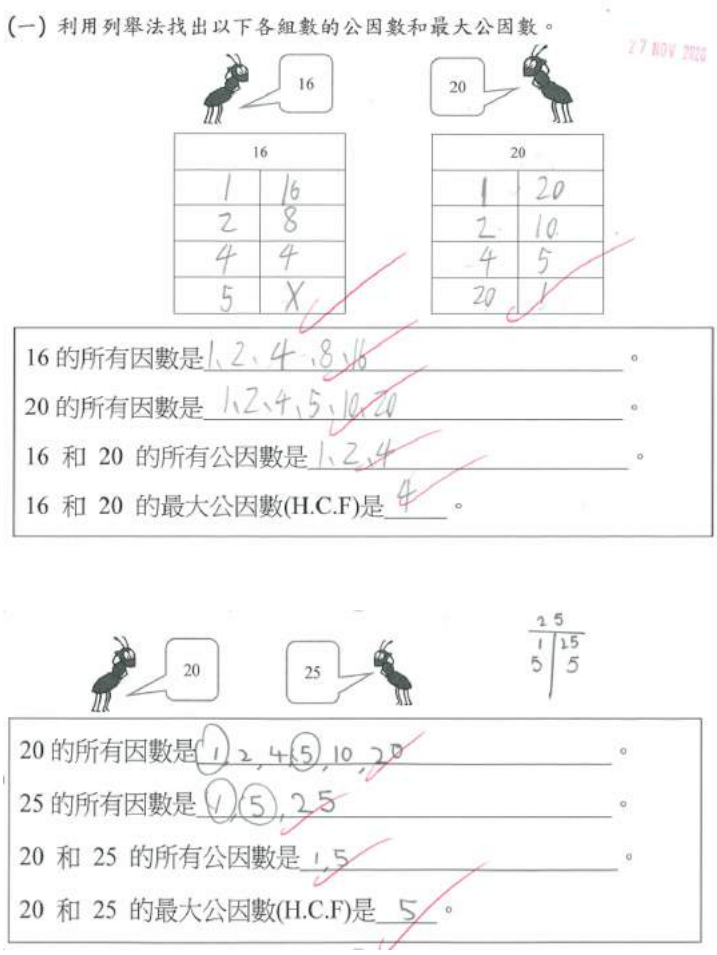
12 的因數有：1,2,3,4,6,12

不論是8還是12，也可以平均排成1行、2行或4行。所以8和12的公因數是1、2和4。大家排成多少行最有效率？當然是排成最大公因數：4行最好！

Teachers helped students develop communication and reasoning skills by referring to terms and definitions to use as part of their shared reasoning process. The term "**最大公因數**", abbreviated as H.C.F. (Highest Common Factor), was introduced based on previous work/task, and the definitions were made explicit: **某數同時是兩個數的因數，便是它們的公因數。而當中最大的公因數，又稱為最大公因數，簡稱**H.C.F.（Highest Common Factor）。**例如8和12的公因數是**1、2、4。**當中的最大公因數（**H.C.F.）**是**4。

STUDENT WORK

Teachers modified the story plot and created new mathematical problems using an idea that the class had explored and studied to help make sense of common factors and highest common factor. There was a rich connection to students' life experiences, while promoting equity and fairness in the learning of mathematics. Teachers guided students to become critical thinkers and problem-solvers through the inquiry process.



(一) 利用列舉法找出以下各組數的公因數和最大公因數。

16

16	
1	16
2	8
4	4
5	X

20

20	
1	20
2	10
4	5
20	1

16 的所有因數是 1、2、4、8、16 。

20 的所有因數是 1、2、4、5、10、20 。

16 和 20 的所有公因數是 1、2、4 。

16 和 20 的最大公因數(H.C.F)是 4 。

20 25

20 的所有因數是 1, 2, 4, 5, 10, 20 。

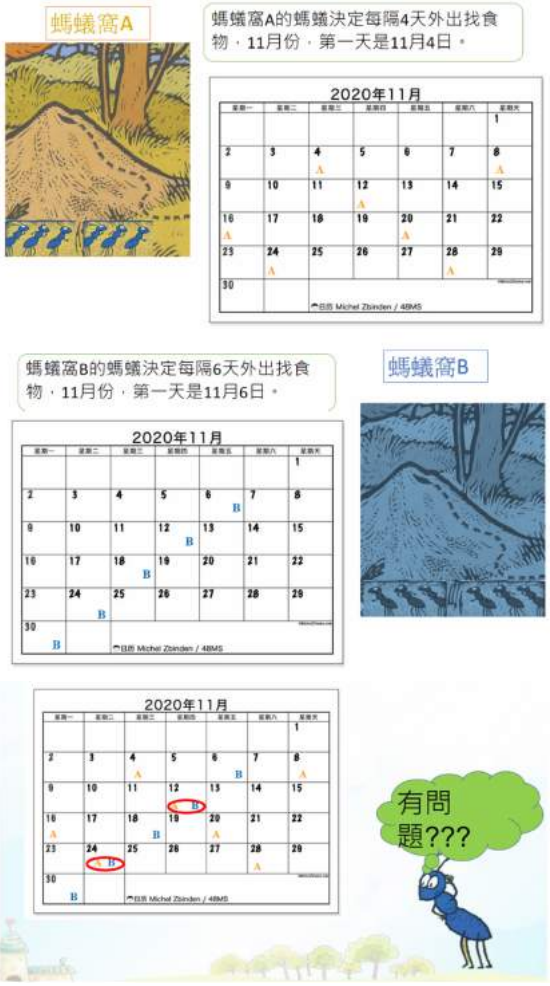
25 的所有因數是 1, 5, 25 。

20 和 25 的所有公因數是 1, 5 。

20 和 25 的最大公因數(H.C.F)是 5 。

COMMON MULTIPLES (公倍數) AND THE LOWEST COMMON MULTIPLE (最小公倍數)

森林裏住了兩群螞蟻，牠們住在不同的蟻窩內，但是常常因爭食物互相打鬥，死傷無數……最後牠們決定向蟻王求助，蟻王提議，兩個螞蟻窩在不同日子出外找食物，減少見面機會……大家都認為這是一個好方法。



Lorem ipsum

那麼出現公倍數的日子，我們會不會再打架？蟻王說：不用怕，出現公倍數的日子，我會在場維持秩序，但是11月份，我第一天出席的日子會是哪一天？

Instead of direct computation, students were given a calendar to work out the multiples of 4 and 6. Teachers ensured that students were constantly engaged in using the vocabulary during their classroom discussion. For example, Ms. Lam asked "**每隔六日去一次, 第三次係… 咁即係六嘅幾多倍**?" Students listed out all the multiples on the calendar and realized the problem in the given situation, where the two groups of ants would meet and fight against each other. One said "**唔對路**" and the other agreed and replied "**12號同24號會撞埋一齊**". Based on students' responses, Ms. Lam revoiced by rephrasing "**12同24係4同6嘅公倍數**". She concluded by saying "**蟻王第一天出席的日子會是12, 係最小公倍數, 英文簡稱**L.C.M."

What started out as a simple idea of multiples and factors develops into a complex network of mathematical ideas, which help students make the connections between them. Teachers played with the story plot to develop ideas that open out other teaching/learning opportunities. The story makes understanding possible at different levels: One level ants line up in row(s); More complex levels highest common factor and lowest common multiple. Such rich connections deepen students' learning of mathematically inter-related concepts through vocabulary. This is a key aspect of mathematical vocabulary instruction, particularly with multilingual students. Teachers created a language-rich classroom environment, emphasizing essential terminology/vocabulary on the PowerPoint, in which students experienced mathematical new vocabulary items when discussing problems in context.

PRIME NUMBERS (質數) AND COMPOSITE NUMBERS (合成數)

牠們都知道100隻螞蟻用因數的方式排隊的事。所以牠們都想提早學習排隊的方式。我們先把不同蟻窩的因數找出來。Students were asked to find the factors of the numbers using array models. Through this task, students understand a prime number has exactly two different factors, the number itself and 1.

Students were asked to sift out all prime numbers up to 100 on the number chart, crossing out the multiplies. Numbers on the chart are primes; those crossed out are composite numbers. When teachers used the same context throughout the unit, students can spend less time to understand the language, and more time making sense of the mathematical content.

CONCLUDING REMARKS

There are many ways for students to model the mathematics they encounter. They can use physical models (lining up in person), visual models, creating a table, etc. When teachers pose problems with a familiar context, students can spend less time trying to understand the language and more time making sense of the mathematical concept. In this example, teachers develop students' problem-solving strategies through the use of array models, providing prompts and sentence starters/-frames, moving from simple problems to more complex ones through the unit of multiples and factors. Teachers also provided ways for students to read, define, label and practise using the mathematical terms so that students can understand how the terms are being used in word problems [in the textbook] and use the terms in the explanations of their problem-solving process.

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