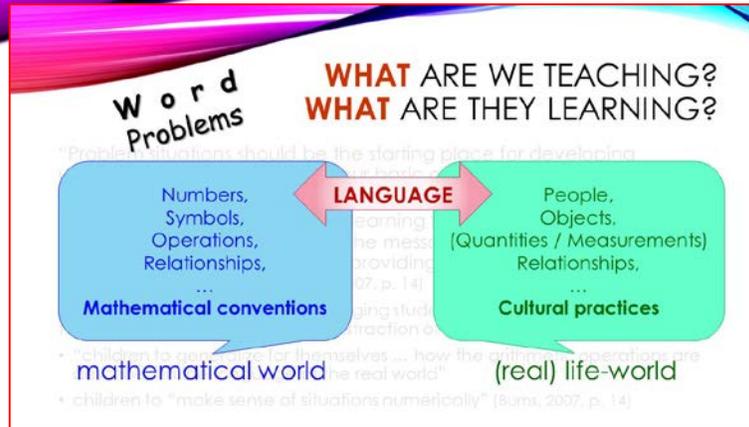


# WORD PROBLEMS

a **headache** in our learning and teaching of mathematics, **not least the classrooms with ethnic minority students**

**Ka Lok Wong**

March 6, 2020



**Word Problems** **PUTTING THE CART BEFORE THE HORSE?**

“A long-standing instructional practice has been to teach students how to add, subtract, multiply, and divide and then, after students have learned to compute, give them word problems to solve. I think of this instructional practice as putting the cart before the horse.”  
 (Marilyn Burns, 2015)

Source: [www.marilynburnsmathblog.com/word-problems-dont-put-the-cart-before-the-horse/](http://www.marilynburnsmathblog.com/word-problems-dont-put-the-cart-before-the-horse/)

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# WHAT ARE “WORD PROBLEMS”?

- word problems?
- application problems?
- 文字題?
- 應用題?

# WHAT ARE “WORD PROBLEMS”?

“文字題”

2 occurrences

“應用題”

28 occurrences

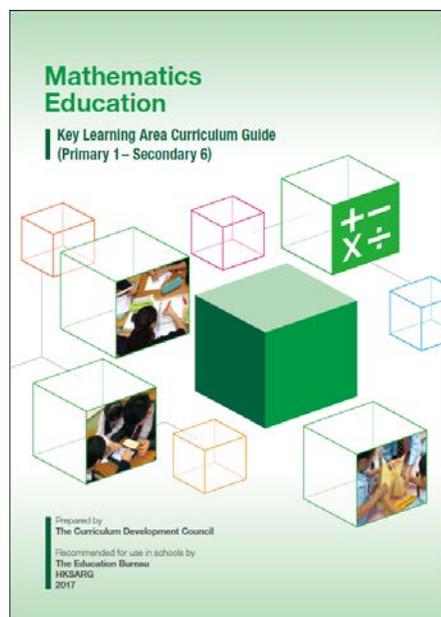


“word problem”

1 occurrence

“application  
problem”

1 occurrence

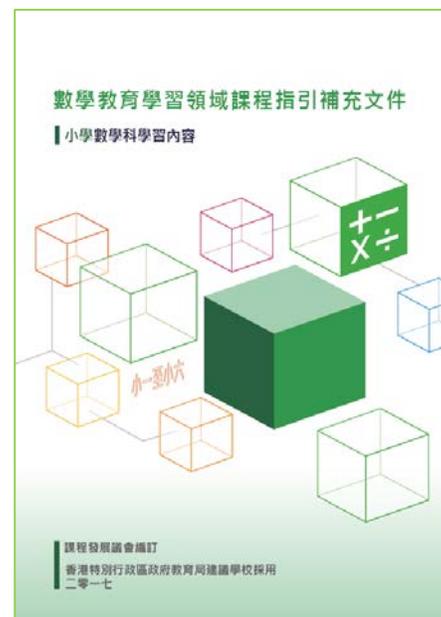


“文字題”

0 occurrence

“應用題”

61 occurrences

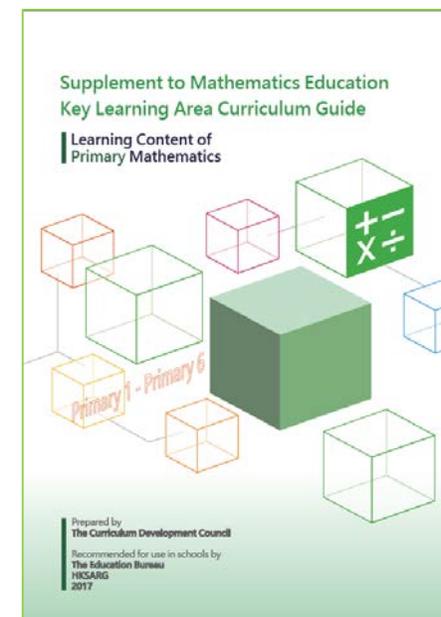


“word problem”

0 occurrence

“application  
problem”

0 occurrence



# WHAT ARE "WORD PROBLEMS"?

- word problems?
  - application problems?
  - story problems?
  - real world problems?
  - ...
- 文字題?
  - 應用題?
  - 故事題?
  - 情境題?
  - .....

Mainly concerned with ...

- application
- "real-world" situations
- situations described in words
- solving problems
- problem solving

Is it a matter of simple translation?  
文字題 = word problem(s) ?

- a convention
- a practice
- a linguistic practice
- a cultural practice





# **DIFFICULTIES** WITH WORD PROBLEMS

When your students tackle a word problem, what do you think are their major difficulties?

(You may choose up to 3 options.)

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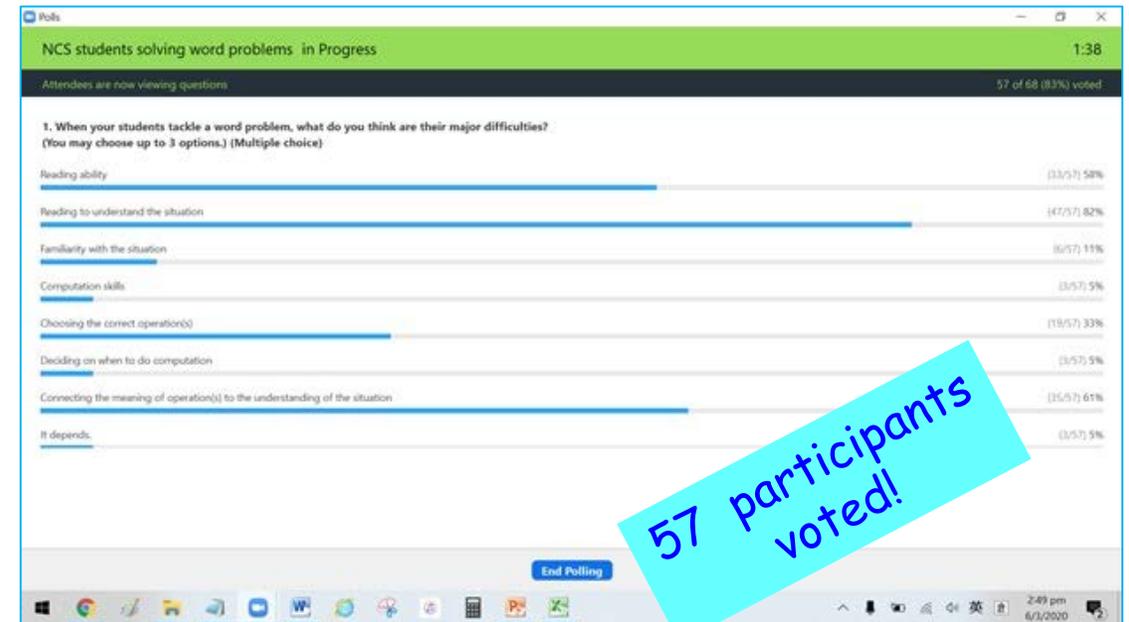
- Reading ability
- Reading to understand the situation
- Familiarity with the situation
- Computation skills
- Choosing the correct operation(s)
- Deciding on when to do computation
- Connecting the meaning of operation(s) to the understanding of the situation
- It depends.

# DIFFICULTIES WITH WORD PROBLEMS

When your students tackle a word problem, what do you think are their major difficulties?

(You may choose up to 3 options.)

- Reading ability
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# DEALING IN HORSES

A man bought a horse for \$50 and sold it for \$60. He then bought the horse back for \$70 and sold it again for \$80. What do you think was the financial outcome of these transactions?



a word problem  
for you

# DEALING IN HORSES\*

A man bought a horse for \$50 and sold it for \$60. He then bought the horse back for \$70 and sold it again for \$80. What *do you think* was the financial outcome of these transactions?

- Lost \$20
- Lost \$10
- Came out even
- Other (describe \_\_\_\_\_ )
- Earned \$10
- Earned \$20
- Earned \$30

\* This is a problem put forward in Burns (2007, p. 14).

# DEALING IN HORSES\*

A man bought a horse for \$50 and sold it for \$60. He then bought the horse back for \$70 and sold it again for \$80. What do you think was the financial outcome of these transactions?

The scenario is easy to understand, although there are some less common words. The calculations involved are not difficult. "Yet deciding precisely what to do isn't obvious to everyone. The difficulty lies in knowing the correct way to connect the arithmetic operations to the situation in order to arrive at a solution." (Burns, 2007, p. 13)

\* This is a problem put forward in Burns (2007, p. 14).

# DIFFICULTIES WITH WORD PROBLEMS

Traditional sequence of teaching/learning:

Numerical symbols → Computational skills → Application problems

Difficulties arising from ... ?

- Reading ability
- Reading to understand the situation
- Familiarity with the situation
- Computation skills
- Choosing the correct operation(s)
- Deciding on when to do computation
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- It depends.

We have come across or even tried **various strategies and approaches.**

However, do they all serve well my own understanding of the usage / purposes of "word problems"?

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**Putting the cart before  
the horse?!**

("backward approach")

(Burns, 2007, p. 13)

# DIFFICULTIES WITH WORD PROBLEMS

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**Should we consider more seriously “Using word problems to develop arithmetic understanding”?**

(Burns, 2007, pp. 13-15)

# Word Problems

## PUTTING THE CART BEFORE THE HORSE?

“A long-standing instructional practice has been to teach students how to add, subtract, multiply, and divide and then, after students have learned to compute, give them word problems to solve. I think of this instructional practice as putting the cart before the horse.”

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- application problems?

- story problems?

- world problems?

- 文字題？

- 應用題？

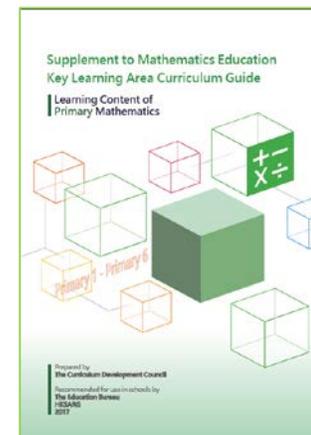
- 故事題？

- 情境題？

- .....

Mainly concerned with ...

- application
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# BASIC STANCE

“ Problem situations should be the starting place for developing understanding of each of the four basic operations of arithmetic ... thereby establishing the need and context for computation skills.

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“Problem situations should be the starting place for developing understanding of each of the four basic operations of arithmetic ... thereby establishing the need and context for computation skills. Children need to see that learning to compute serves a purpose – for solving problems. Too often, the message is reversed, and children see word problems as a way of providing computation practice, and a mysterious way at that.” (Burns, 2007, p. 14)

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- Does it make sense?
- Does it work (in real classroom / curriculum practice)?
- **What is yours?**

# BASIC STANCE

“Problem situations should be the starting place for developing understanding of each of the four basic operations of arithmetic ... thereby establishing the need and context for computation skills. Children need to see that learning to compute serves a purpose – for solving problems. Too often, the message is reversed, and children see word problems as a way of providing computation practice, and a mysterious way at that.” (Burns, 2007, p. 14)

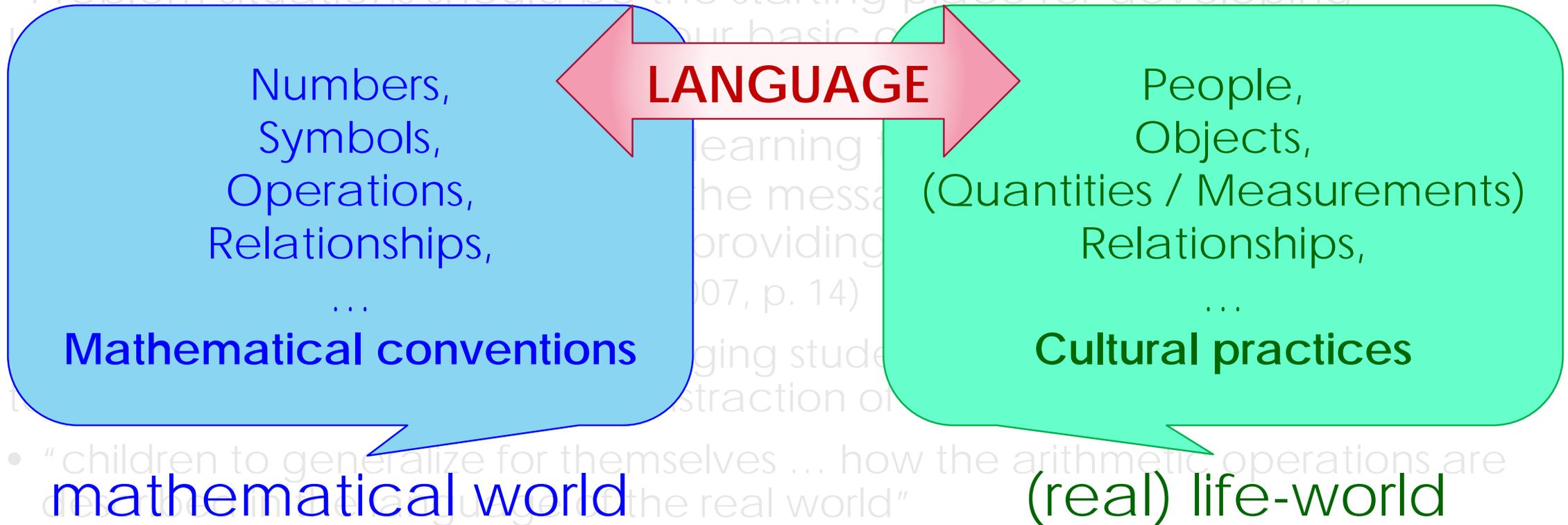
→ Presenting / teaching / encouraging students to discuss (and find solutions to) word problems, “without the distraction of numerical symbols”, develops ...

- “children to generalize for themselves ... how the arithmetic operations are described in the language of the real world”
- children to “make sense of situations numerically” (Burns, 2007, p. 14)

Word  
Problems

WHAT ARE WE TEACHING?  
WHAT ARE THEY LEARNING?

"Problem situations should be the starting place for developing



• "children to generalize for themselves ... how the arithmetic operations are

• children to "make sense of situations numerically" (Burns, 2007, p. 14)

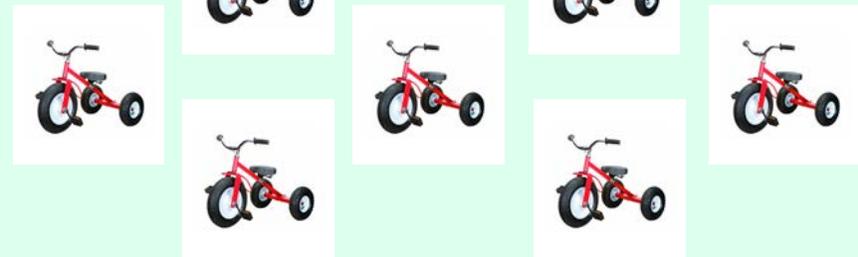
# WHAT ARE WE TEACHING? WHAT ARE THEY LEARNING?

an example

(considered by Burns, 2015)

There are 7 tricycles. How many wheels are there altogether?

$$7 \times 3 = ?$$



mathematical world

(real) life-world

- “children to generalize for themselves ... how the arithmetic operations are described in the language of the real world”

- children to “make sense of situations numerically” (Burns, 2007, p. 14)

Video-illustration of the lesson at YouTube: <https://youtu.be/GEIYcBzQljU>

**Any ways** to bring the mathematical symbols closer to the real world?

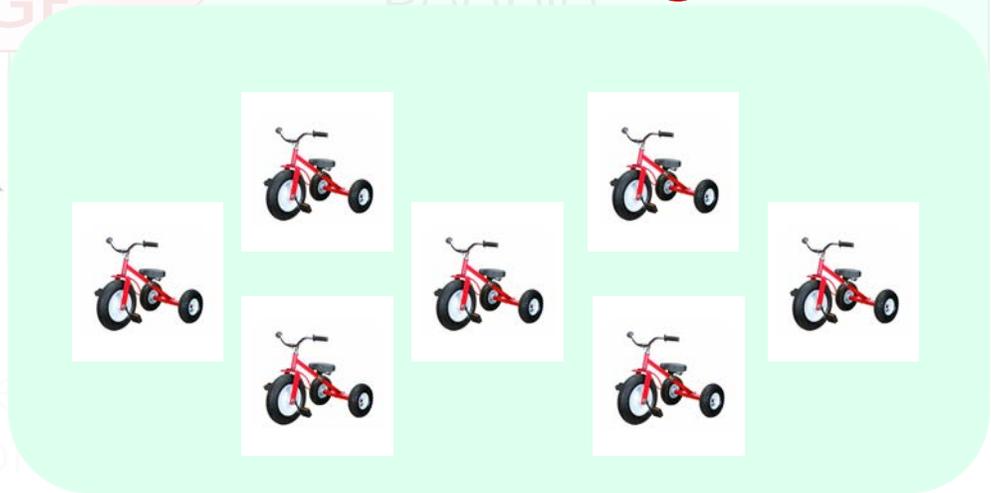
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**Any ways** to represent the real world by more mathematical conventions ?

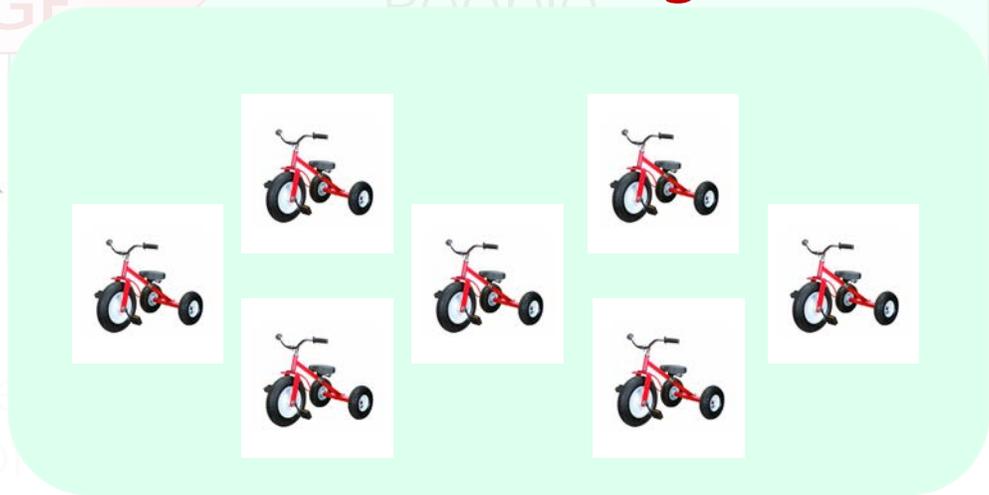
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**Any ways** to bring the mathematical symbols closer to the real world?

HOW  
HOW

**Any ways** to represent the real world by more mathematical conventions?

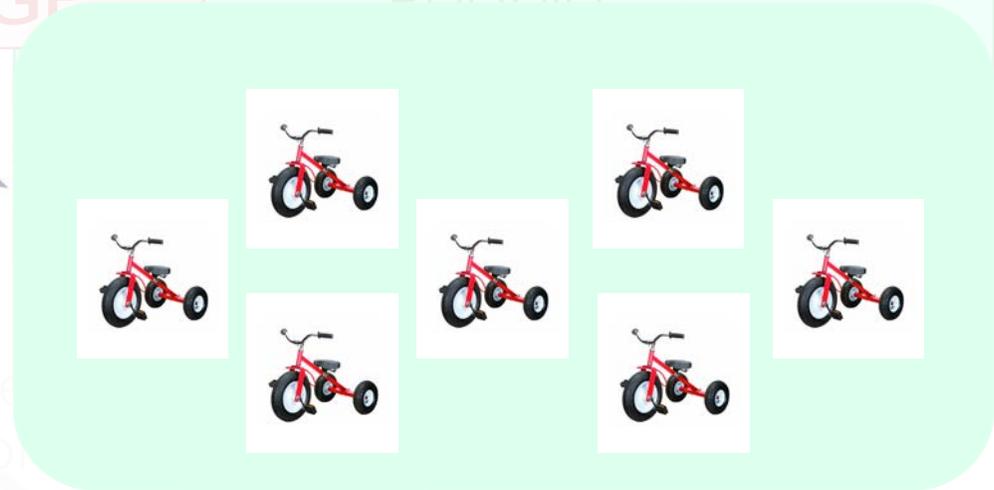
?  
?

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mathematical world

(real) life-world



Any ways to bring the mathematical symbols closer to the real world?

HOW  
HOW

Any ways to represent the real world by more mathematical conventions?

?  
?

an example

(considered by Burns, 2015)

Estimate

- √10
- √20
- √30
- √40
- 50
- 60
- 70
- 80
- 90
- 100
- more than 100

Equation:  $7 \times 3 = \square$

Problem: There are 7 tricycles. How many wheels are there altogether?

Figuring

- 1.
2. 3, 6, 9, 12, 15, 18, 21
3.  $3 \times 3 = 9$  (3 tricycles).  
 $3 \times 3 = 9$  (3 more tricycles)  
 $9 + 9 = 18$  (6 tricycles)  
 $18 + 3 = 21$
- 4.

$7 \times 3 = 21$



Multiplication

Estimate

- 10
- 20
- 30
- 40
- 50
- 60
- 70
- 80
- 90
- 100

Equation:  $8 \times 4 = \square$

Problem:

Figuring:



mathematical world

(real) life-world

# WHAT ARE WE TEACHING? WHAT ARE THEY LEARNING?

an example

(considered by Burns, 2015)

There are 7 tricycles. How many wheels are there altogether?

$$7 \times 3 = ?$$

Practising with problems  
after problems, focusing  
on the mathematical  
procedures connecting  
with the words!?

mathematical world

(re) e-world

- “children to generalize for themselves ... [and] to be able to make a judgement of the real world”

- children to “make sense of situations numerically” (Burns, 2007, p. 14)

Video-illustration of the lesson at YouTube: <https://youtu.be/GEIYcBzQlJU>

# BEYOND WORD PROBLEMS

## another example

(provided by Burns, 2007, p. 17)

1. If  $A = \$0.01$ ,  $B = \$0.02$ ,  $C = \$0.03$ , and so on, what is the value of your first name?
2. Using this alphabet system, one of the days of the week is worth exactly \$1.00. Which one is it?
3. Find other words that are worth exactly \$1.00.

approaches, strategies, ... working habits,  
different kinds of reasoning, ...

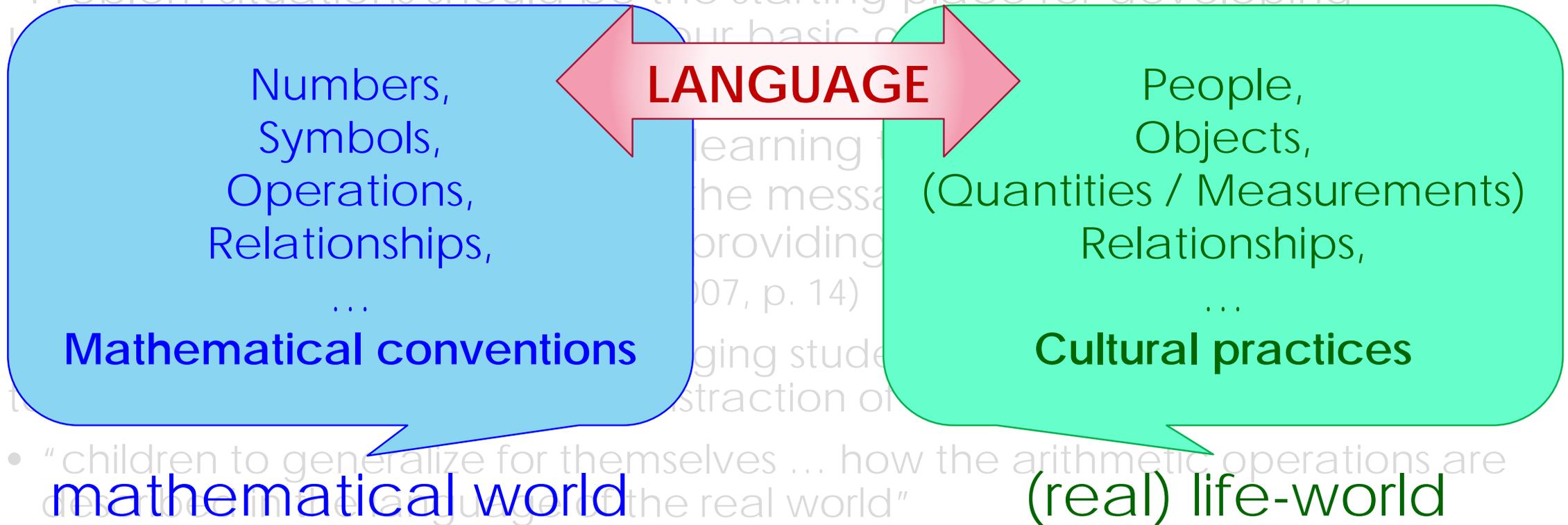
a lot of arithmetic  
practice

→ mainly concerned with **PROBLEM SOLVING** on top of **COMPUTATION**

Word  
Problems

WHAT ARE WE TEACHING?  
WHAT ARE THEY LEARNING?

"Problem situations should be the starting place for developing



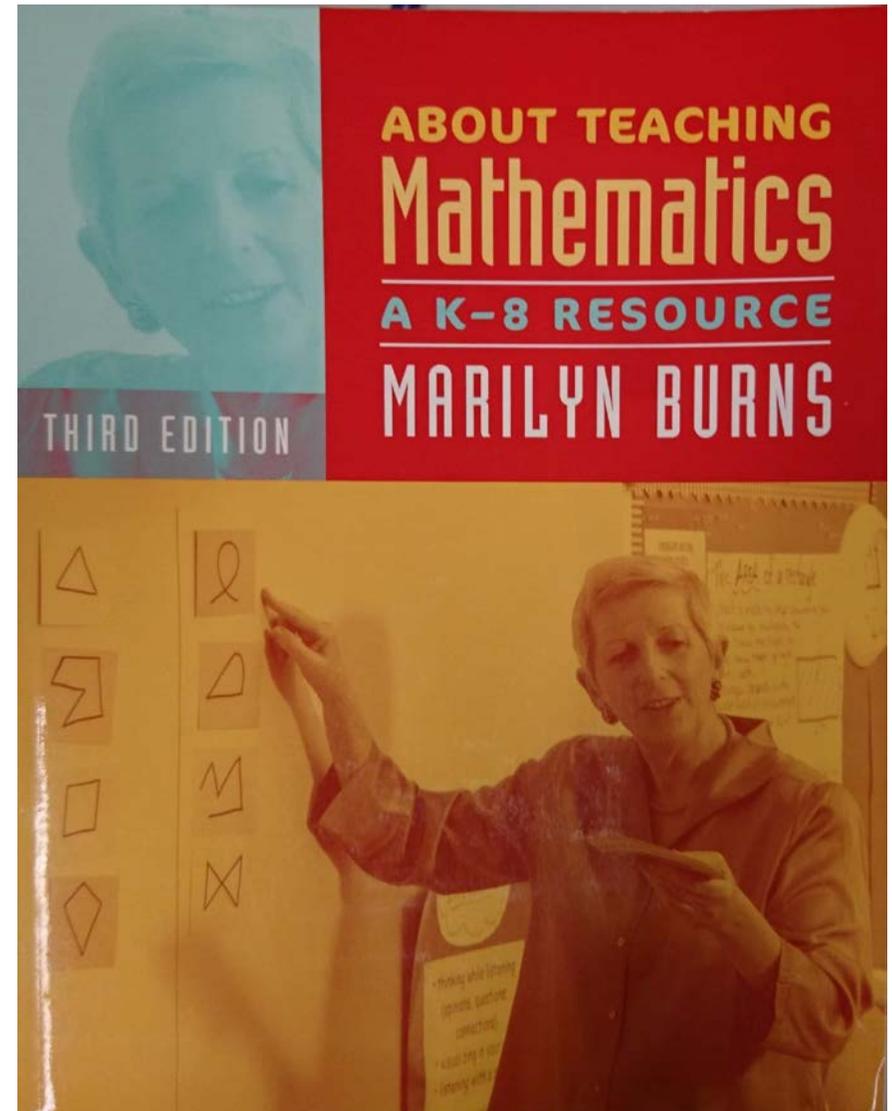
• "children to generalize for themselves ... how the arithmetic operations are described in the language of the real world"

• children to "make sense of situations numerically" (Burns, 2007, p. 14)

my conceptual picture of how Burns' suggestions work in learning & teaching of mathematical word problems

# ABOUT TEACHING MATHEMATICS

Burns, M. (2007). *About teaching mathematics: A K-8 resource* (3<sup>rd</sup> Edn.). Sausalito, CA: Math Solutions Publications.

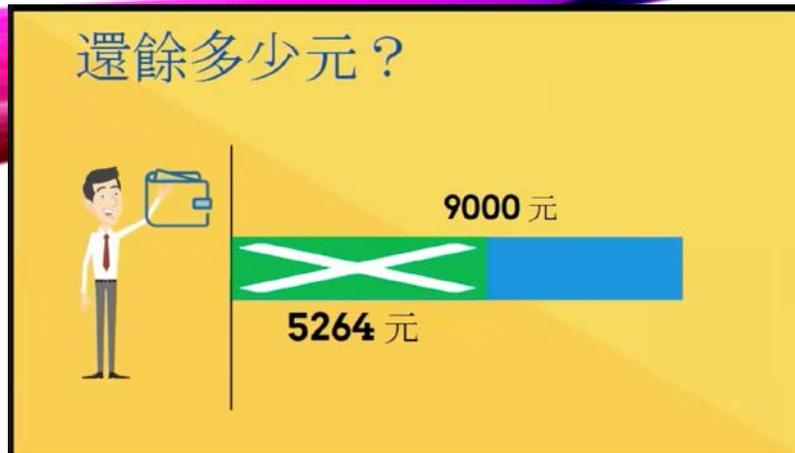


# WORD PROBLEMS

So much a headache that cannot be resolved easily!

*... the above sharing is mostly based on*

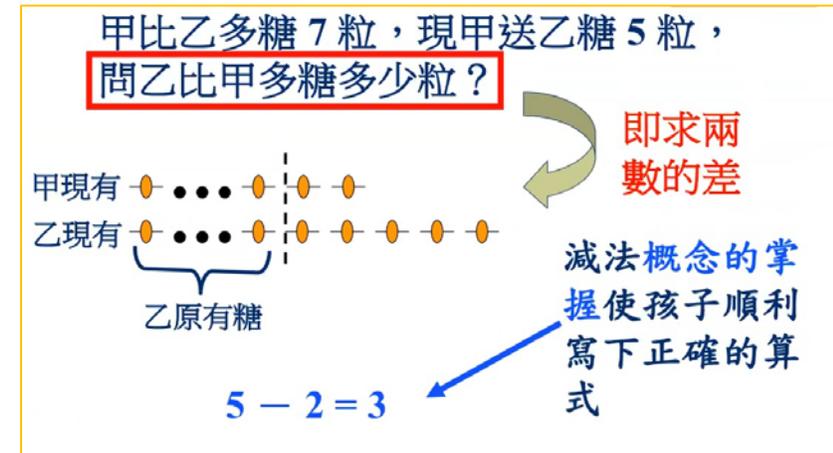
**MY Reading of Marilyn Burns**



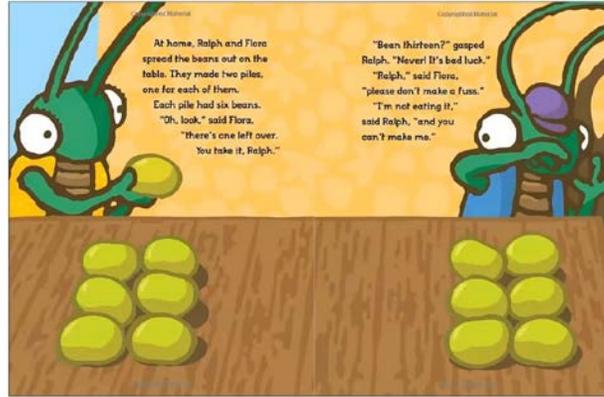
# WORD PROBLEMS

**YOUR Reading** of the strategies and approaches suggested here ...  
under different pedagogical themes

How would you understand them?



Bean Thirteen

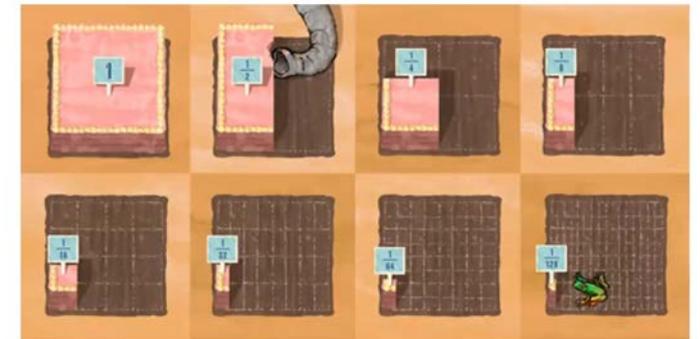


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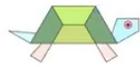
How would you understand them?

Compare Unit Fractions



After 10 minutes, how far is the hare from the "finish line"?

How long does it take for the hare to complete a 15km race?



I can travel 6 km in 20 hours



My speed is 60 km/h



# WORD PROBLEMS

**YOUR Reading** of the strategies and approaches suggested here ... under different pedagogical themes

How would you understand them?

3 apples and a pack of candy (\$10) cost \$34

If we didn't buy candy, 3 apples cost (\$34 - \$10)

One apple costs

$$3x + 10 = 34 \circ$$

$$3x = 34 - 10 \circ$$

$$3x = 24 \circ$$

$$x = \frac{24}{3} \circ$$

$$x = 8 \circ$$

甲原有15元，他用了6元去買3枝鉛筆後。

甲原有15元，他用了6元買筆之後，共有筆5枝。

甲原有筆4枝，他用了6元買筆之後，共有筆5枝。

甲原有筆5枝，他用了6元買筆之後，餘下9元。

- 甲原有筆幾枝？
- 甲原有幾元？
- 甲買了筆幾枝？
- 筆1枝售價幾元？
- 甲用去幾元？

2. 認真閱讀題目，找出不能回答問題的原因，把代表原因的英文字母圈起來。

A. 「資料」不足  
 B. 「資料」與「問句」沒有關係  
 C. 沒有「問句」

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# WORD PROBLEMS

**YOUR Reading** of the strategies and approaches suggested here ... under different pedagogical themes

How would you understand them?

① 王太太早上買了梨子 12 個。  
 王太太早上和下午共買了梨子多少個？ (A/B/C)

② 姊姊有洋娃娃 6 個，  
 弟弟有機械人 18 個。 (A/B/C)

③ 花園內有紅花 18 朵、  
 黃花 15 朵  
 和 白花 12 朵。  
 爸爸今年多少歲？ (A/B/C)

④ 志軒有英文圖書 18 本  
 和 中文圖書 26 本。  
 志軒有圖畫簿多少本？ (A/B/C)



# WORD PROBLEMS

**YOUR Reading** of the strategies and approaches suggested here ... under different pedagogical themes

How would you understand them?

**writing / posing their own word problems**