

MOVING BEYOND ALGORITHM THROUGH PROBLEM SOLVING

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OBJECTIVES

- Recognise problem solving as a tool for learning maths and as a goal of learning in itself
- Caring diversity in problem solving activities
- Analyse problem-solving activities

THREE PERSPECTIVES

- **Problem solving as a goal:** Learn about how to problem solve.
- **Problem solving as a process:** Extend and learn math concepts through solving selected problems.
- **Problem solving as a tool for applications and modelling:** Apply math to real-world or word problems, and use mathematics to model the situations in these problems.

THE PROBLEMS

- Contextual problems offering opportunities for students to develop informal solution strategies, and are used to support mathematical concept building
- The context may even be rather unrealistic or within mathematics, if concept development requires it
- The contextual problem must be experienced as a real problem by the students

(Doorman, Michiel, Drijvers. et al, 2007)

- Good problem solving activities provide an entry point that allows all students to be working on the same problem.

Suppose 39 students want to share 5 candy bars fairly.
How much can each student get?

Leo: That's 5 divided by 39, and we decided last year that you **can't divide a bigger number into a smaller number**.

Anthony: I think that $39 \div 5$ will be 7 remainder 4, but I think that $5 \div 39$ will make **a decimal number**.

Jackson: I think that you will end up with a fraction of a number because, well, because 5 and 39—you can't divide 5 by 39 equally. I think it's going to be **a number below 0**.

After some further discussion about which notation ($39 \div 5$ or $5 \div 39$) actually represents the situation in this problem and what sorts of numbers might be possible answers (e.g., fractions, decimals, remainders, “smaller numbers”)

Suppose 39 students want to share 5 candy bars fairly.
How much can each student get?

Mitchell: So if each kid was going to get equal shares, they would have to cut the five candy bars into little equal pieces.

Teacher (MaryAnn): Can you name those equal pieces?

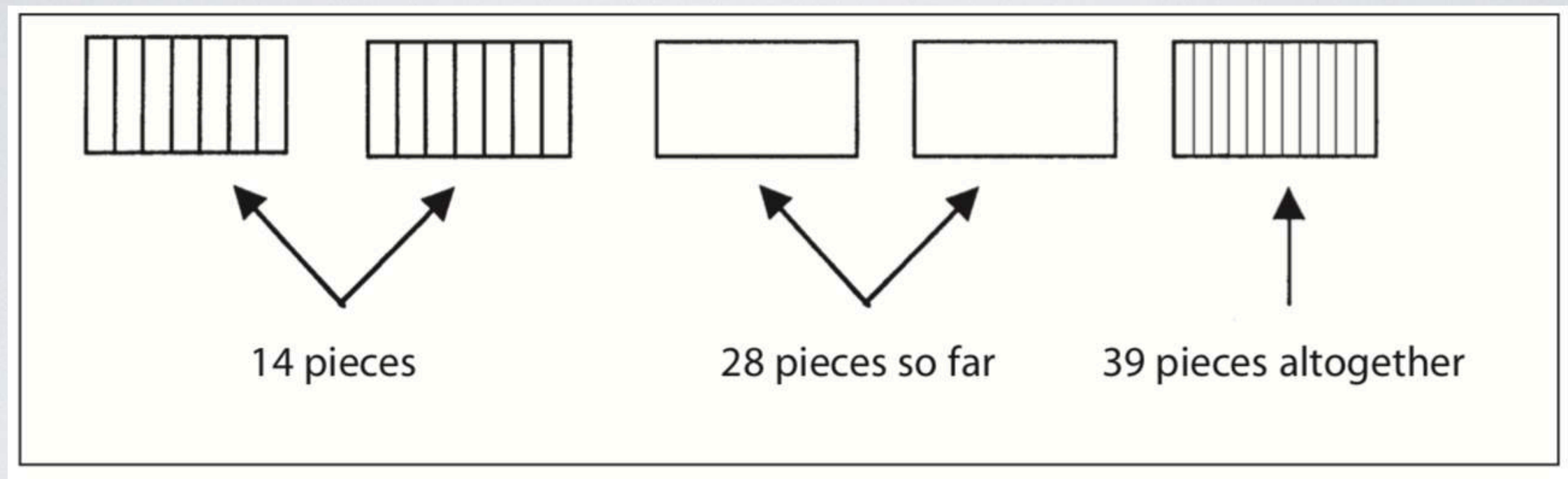
Mitchell: They might be candy bars.

Teacher: Can you name the fraction that they might be?

Teacher: How many people think that you can do the problem $5 \div 39$?
How many think no, you can't?

The results are yes, 13; no, 15.

After a pause, **Leo** says that he wants to change his no to a yes.



Cynthia quickly responds that Leo's representation cannot be correct because it does not yield equal shares. "That's a problem," she says.

Laila: If I cut each of the five candy bars into thirty-nine pieces and then give each kid one piece from each candy bar, you could have each kid have five-thirty-ninths of a candy bar.

After further discussion, most of the class seems convinced that Laila has proposed a valid solution to the problem

BENEFITS OF TEACHING THROUGH PROBLEM SOLVING

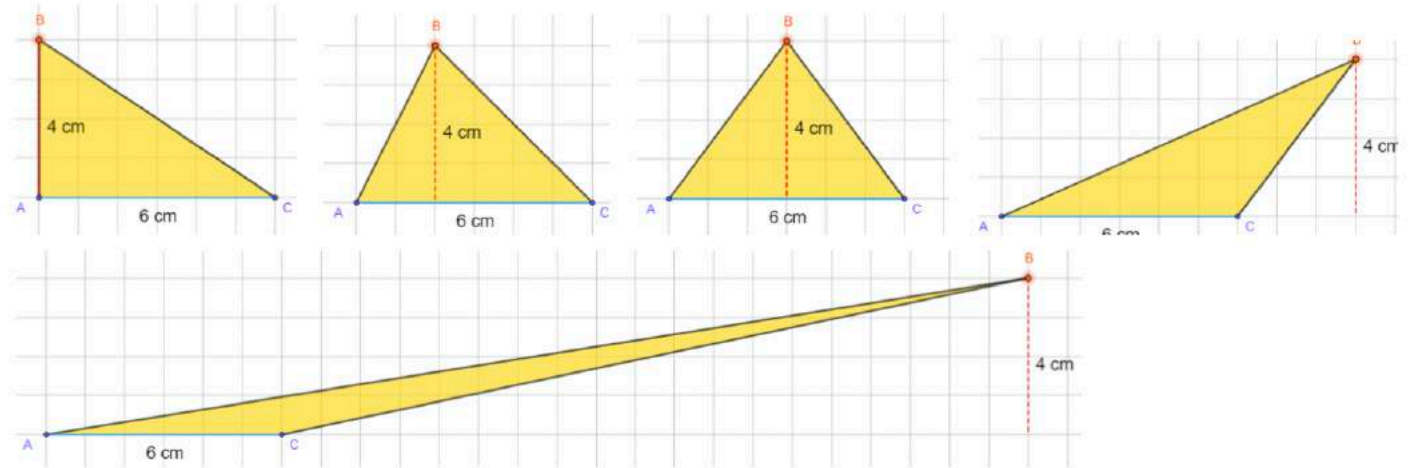
- **Opportunities** for exploring, discussing, experimenting with, and attempting to make sense of mathematical ideas
- **Confident** feeling that ideas make sense
- Promotes **understanding**
- Helps **memory**
- Enhances **Transfer**
- Become **autonomous** learners

STORY: AREA OF POLYGON

- A classroom with NCS students
- Promoting problem solving activities

LEARNING TRAJECTORY: AREA OF POLYGON

1. **Right-angled triangle** (from rectangle)
2. Formula of the area of the right-angled triangle (combine/dissect the triangle)
Is it essential to dissect the triangle?
3. Triangle with the same base and same height.
From right-angled triangle to irregular triangle.
From dotted-square paper to plane figure
Use addition / subtraction?



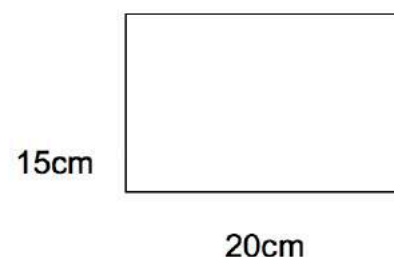
4. Introduce the height of the triangle and the relationship between base and height.
5. By induction, we get the formula of the right-angled triangle can be used for **any kinds of triangle** with the same base and height.
6. Find the area of the triangle without the help of square-dotted paper.
7. Bisect the **parallelogram** into 2 identical triangles. Find the base and the height of the triangle and derive the formula.
By dissecting, rectangle > parallelogram.
By combining, 2 triangles AND a parallelogram > rectangle.
8. Provide different kinds of parallelograms.
9. Use the same principle to introduce the area of **trapeziums**.

AREA OF TRIANGLE

(First version)

1. 下圖中的長方形面積是多少 cm^2 ?

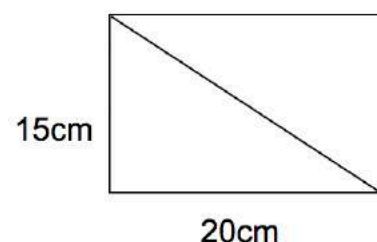
答案：_____ cm^2



大家還記得計算長方形面積的公式嗎？

長方形面積= _____ X _____

2. 下圖是兩個完全一樣的三角形，它們合成一個長方形，你知道一個三角形的面積是多少 cm^2 ?



長方形的面積是_____ cm^2

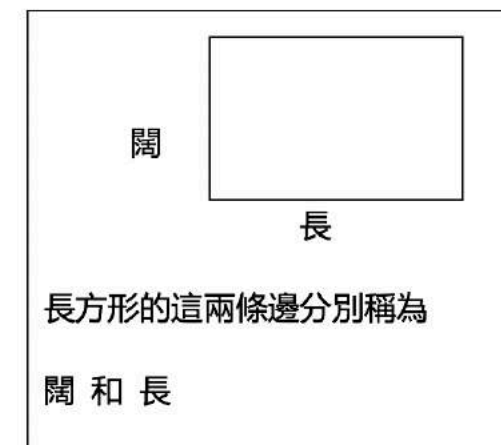
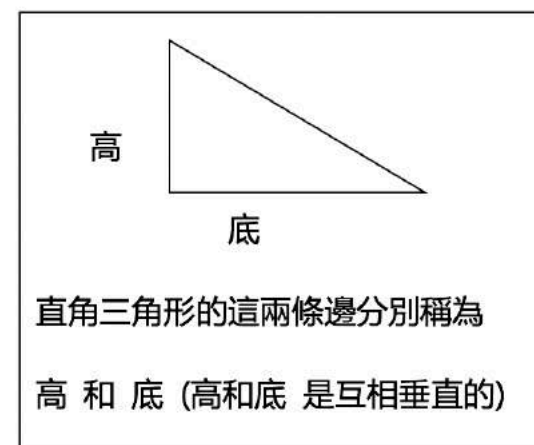
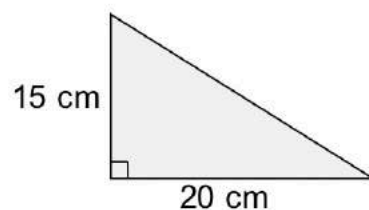
一個三角形的面積是_____ cm^2

如何找出答案呢？

a. 把第一題的答案_____

b. 為什麼？(請舉手告訴老師)

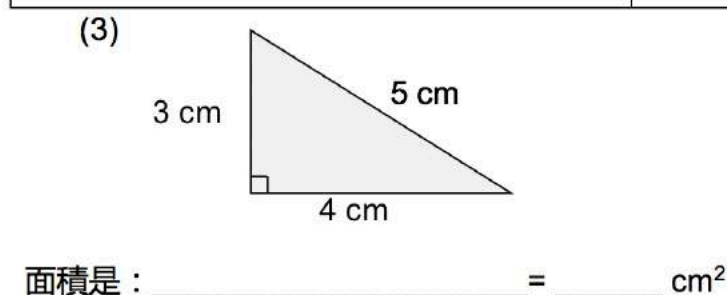
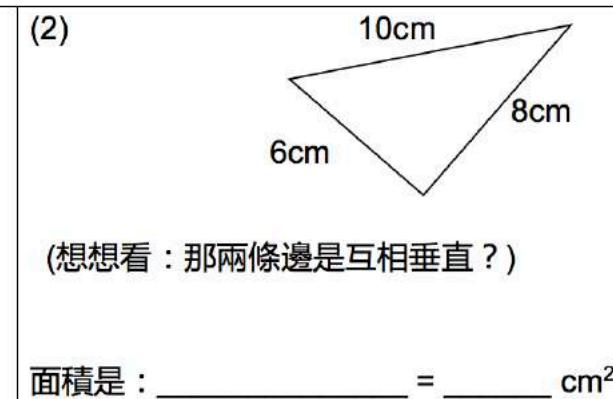
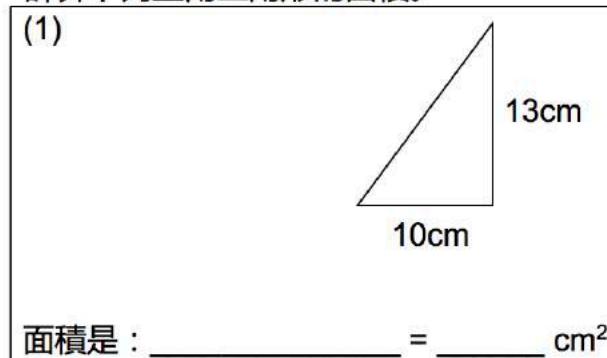
大家想想可怎樣找出三角形面積的公式嗎？



三角形面積的公式

三角形面積 $\frac{\text{底} \times \text{高}}{2}$

計算下列直角三角形的面積。



提示：

如果底長是 4 cm ,

那高度是_____ cm

AREA OF TRIANGLE

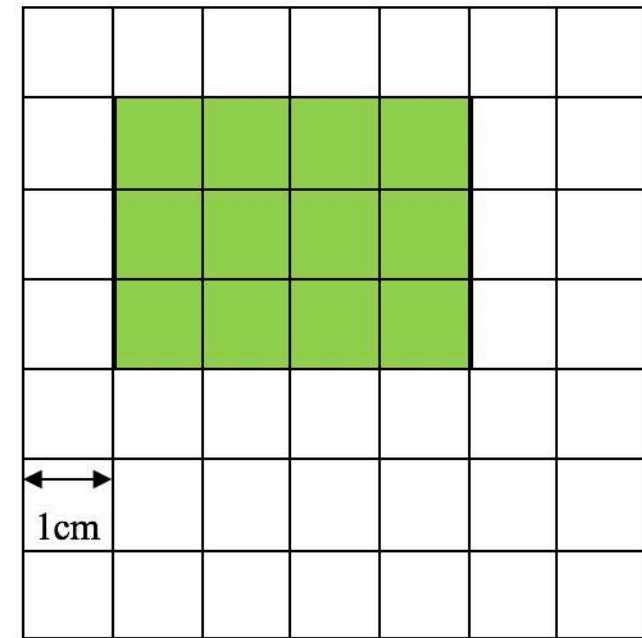
(Second version)

1. Find the area of the rectangle on the right.

Answer : _____ cm²

Do you remember?

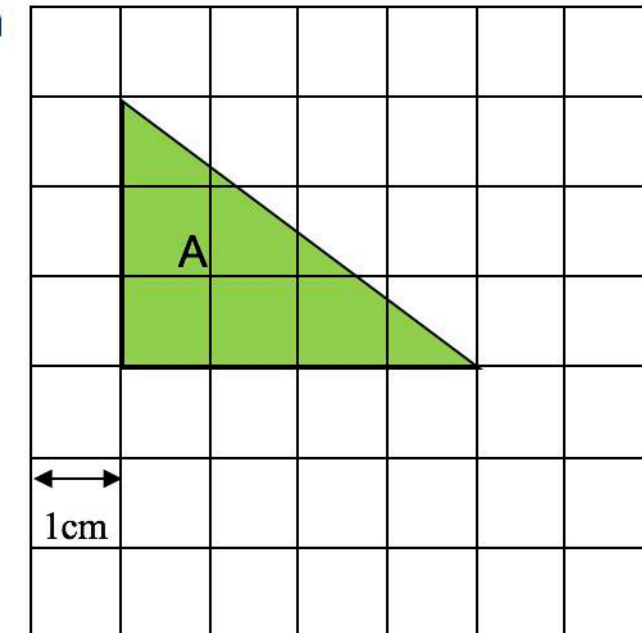
Area of a rectangle = _____ X _____



2. Find the area of the right-angled triangle on the right.

Answer : _____ cm²

Show your work here.



Do you know how to find the area of a right-angled triangle?

Area of a right-angled triangle =

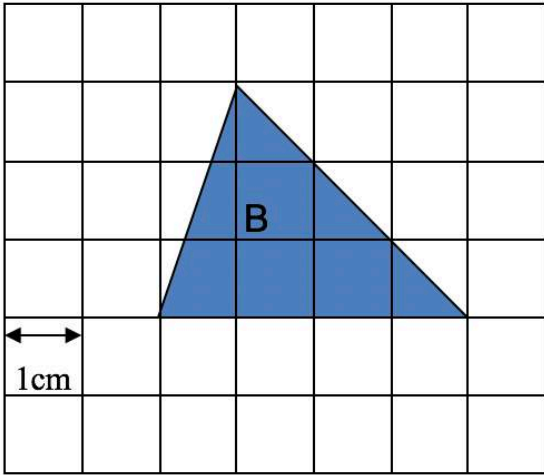
AREA OF TRIANGLE

(Second version)

3. Find the area of the triangle on the right.

Answer : _____ cm²

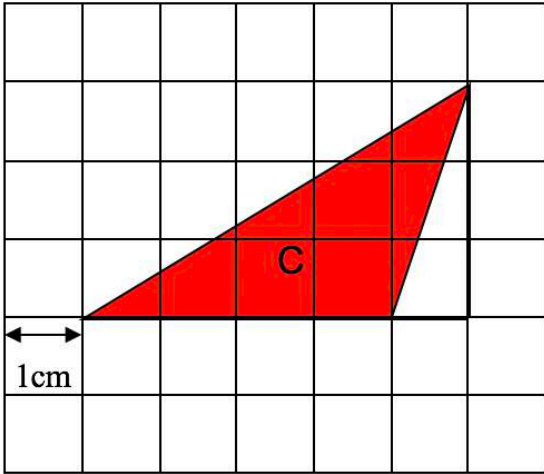
Show your work here.






4. Find the area of the triangle on the right.

Answer : _____ cm²

Show your work here.



5. Base on the results from Q2 to Q4, fill in the table below.

Triangle	Base (cm)	Height (cm)	Area (cm)
A 			
B 			
C 			

6. Do you know how to find the area of a triangle?

Area of a triangle =

AREA OF PARALLELOGRAM

1. Find the area of the parallelogram below.

Hint: we know:

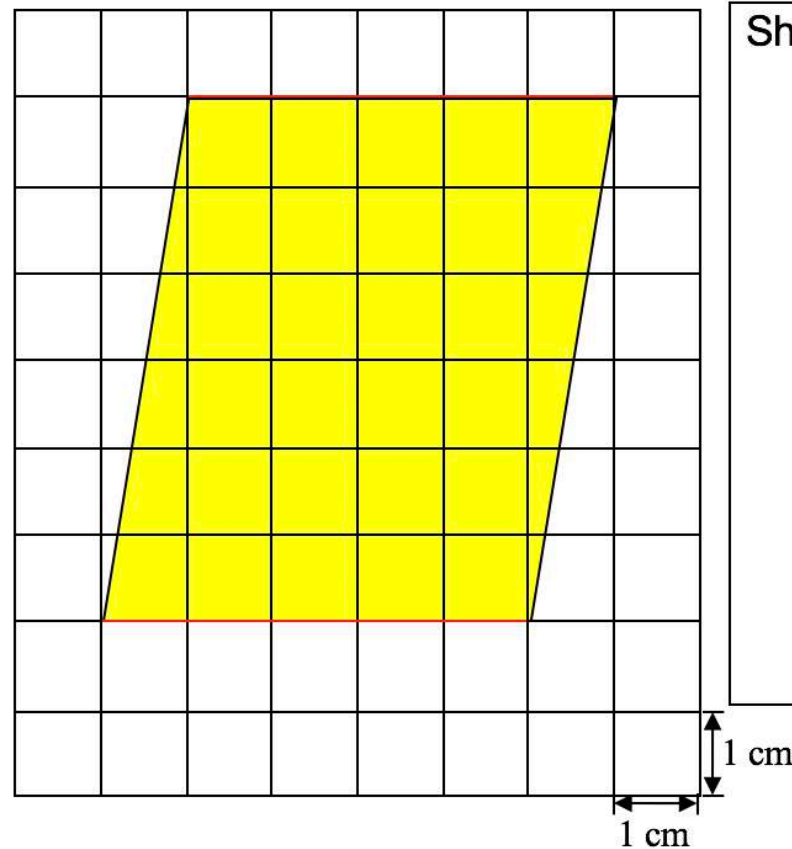
Area of rectangle = Length X Width



Area of square = Length of one side X Length of one side



Area of triangle = $\frac{\text{Base X Height}}{2}$



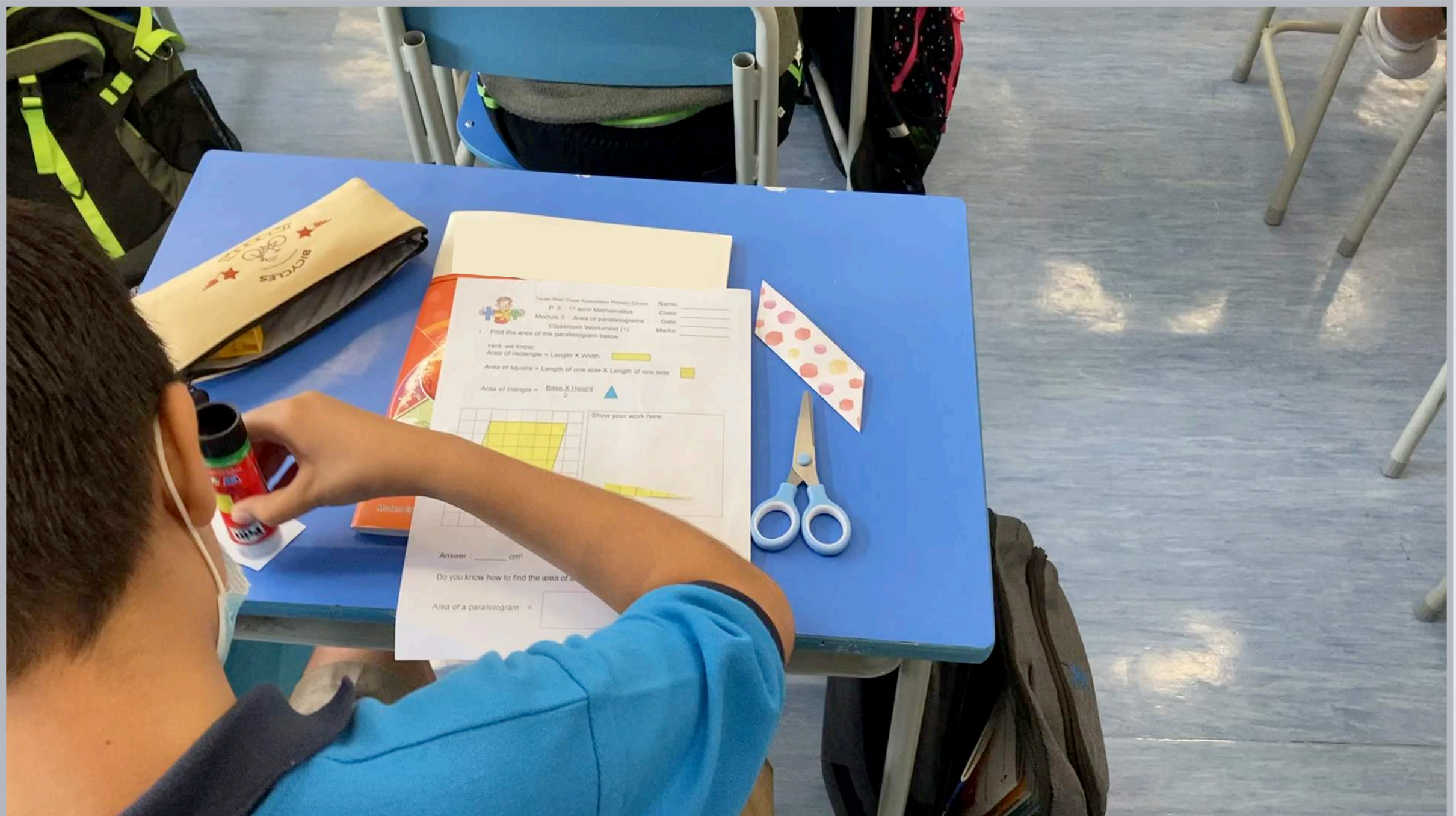
Show your work here.

Answer : _____ cm²

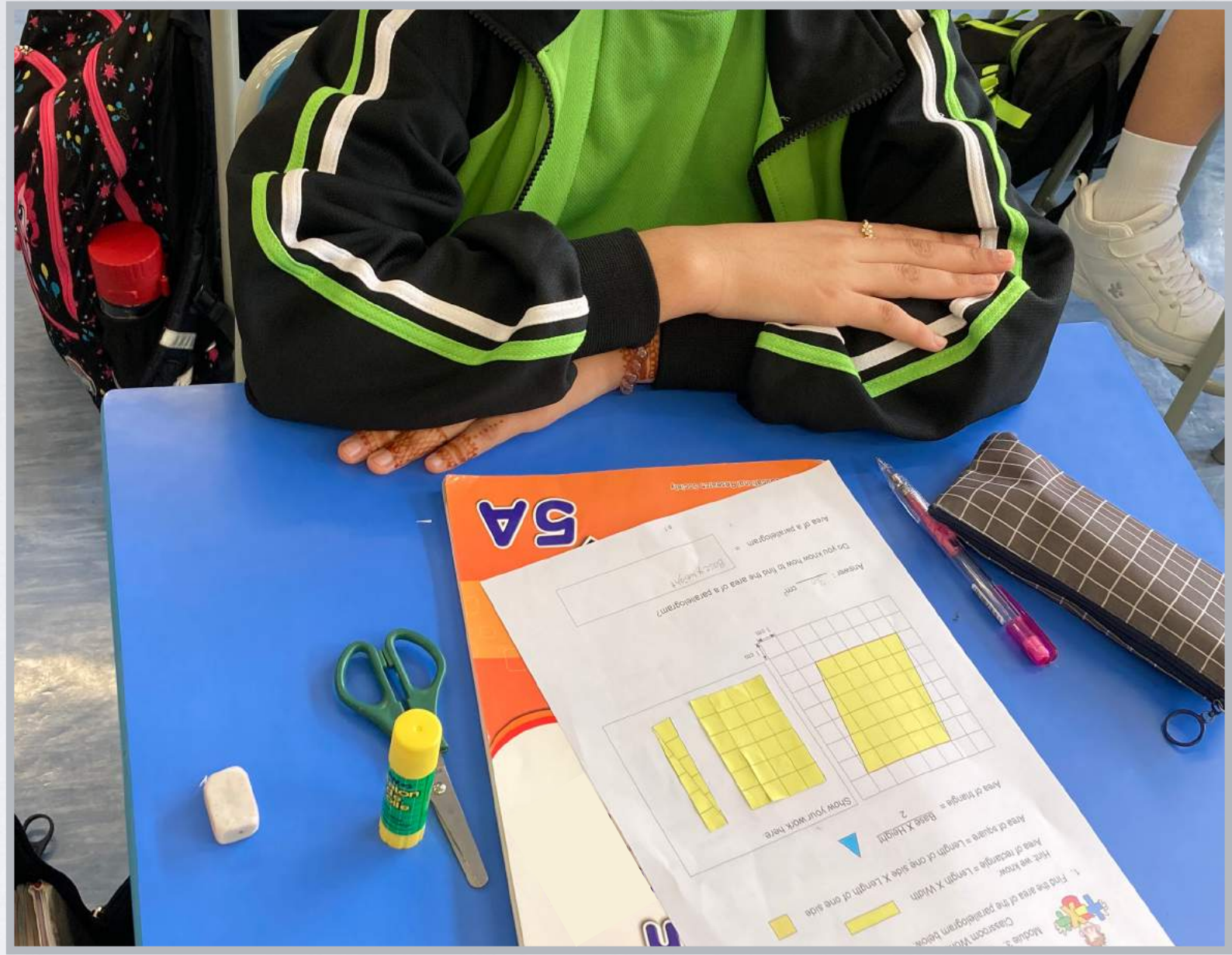
Do you know how to find the area of a parallelogram?

Area of a parallelogram =

AREA OF PARALLELOGRAM

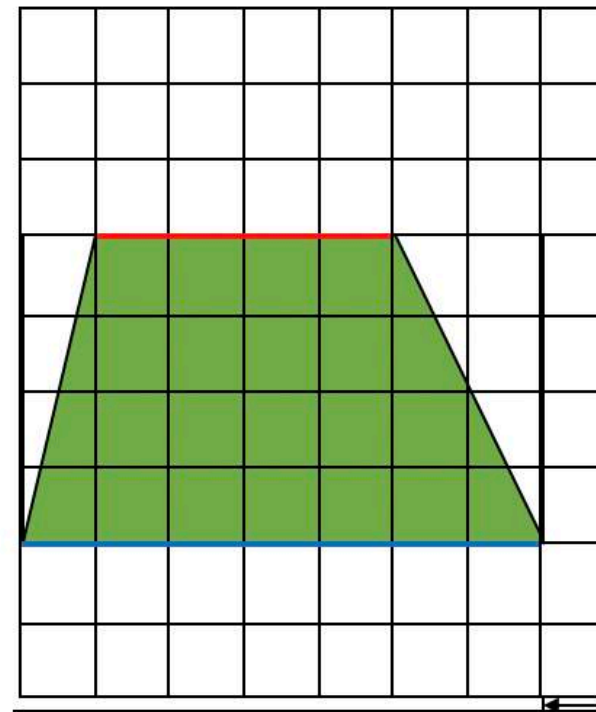






AREA OF TRAPEZIUM

1. Find the area of the trapezium below.



Hint:

Area of parallelogram = Base X Height

Area of triangle = $\frac{\text{Base X Height}}{2}$

Show your work here.

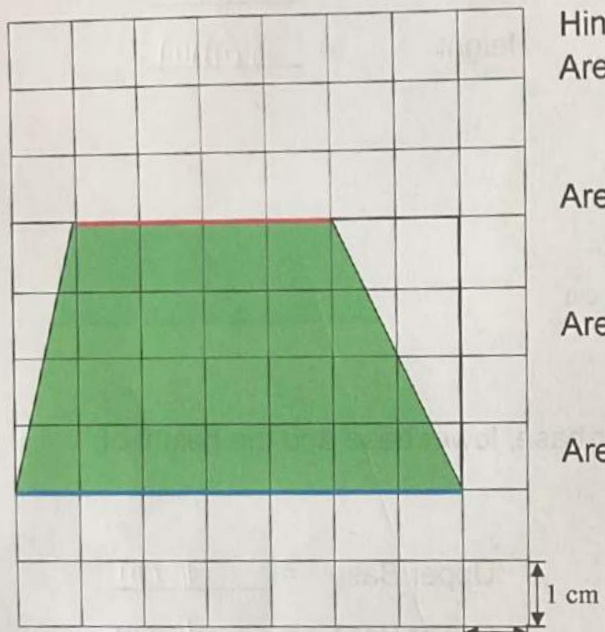
Answer : _____ cm²

Do you know how to find the area of a trapezium?

Area of a trapezium =

AREA OF TRAPEZIUM

1. Find the area of the trapezium below.



Hint:

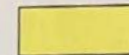
Area of parallelogram = Base X Height



Area of triangle = $\frac{\text{Base X Height}}{2}$



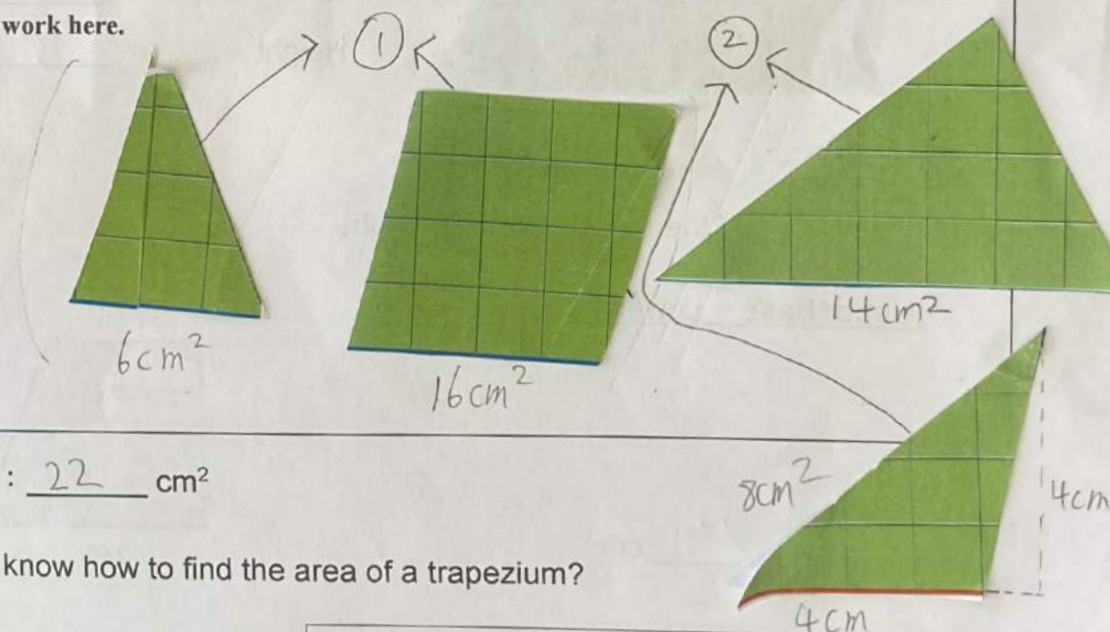
Area of rectangle = Length X Width



Area of square = Length of one side X
Length of one side



Show your work here.

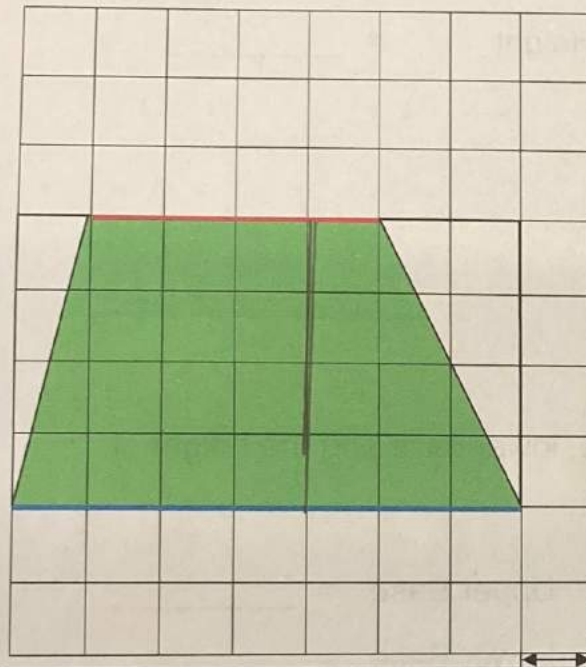


Answer : 22 cm²

Do you know how to find the area of a trapezium?



1. Find the area of the trapezium below.



Hint:

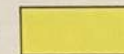


Area of parallelogram = Base X Height

Area of triangle = $\frac{\text{Base X Height}}{2}$



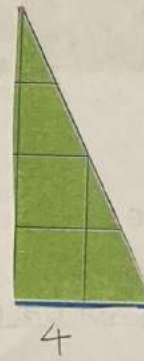
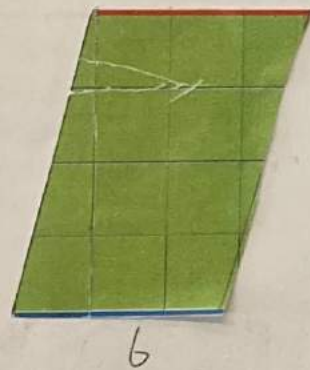
Area of rectangle = Length X Width



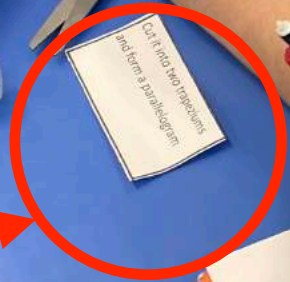
Area of square = Length of one side X
Length of one side

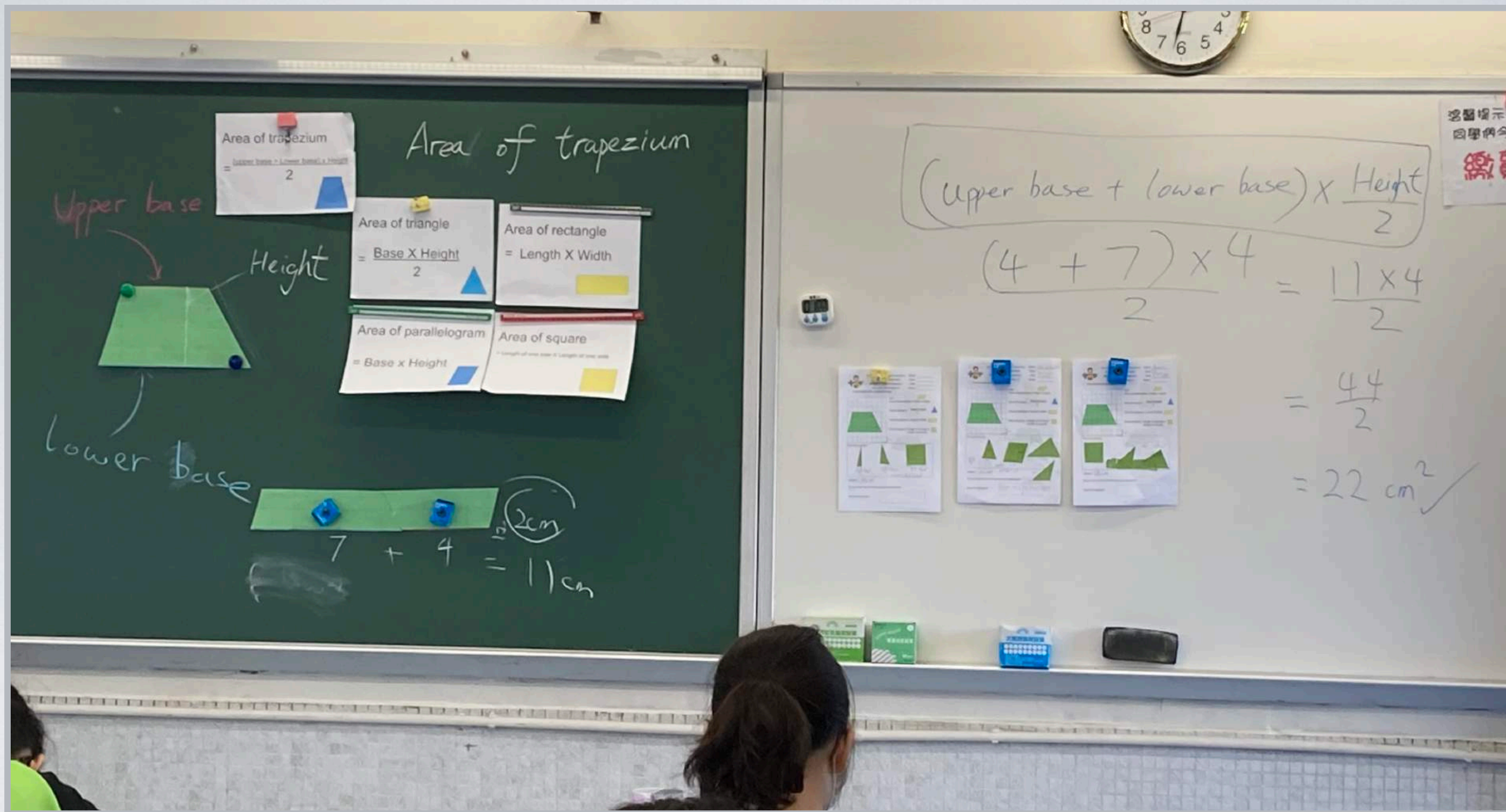


Show your work here.



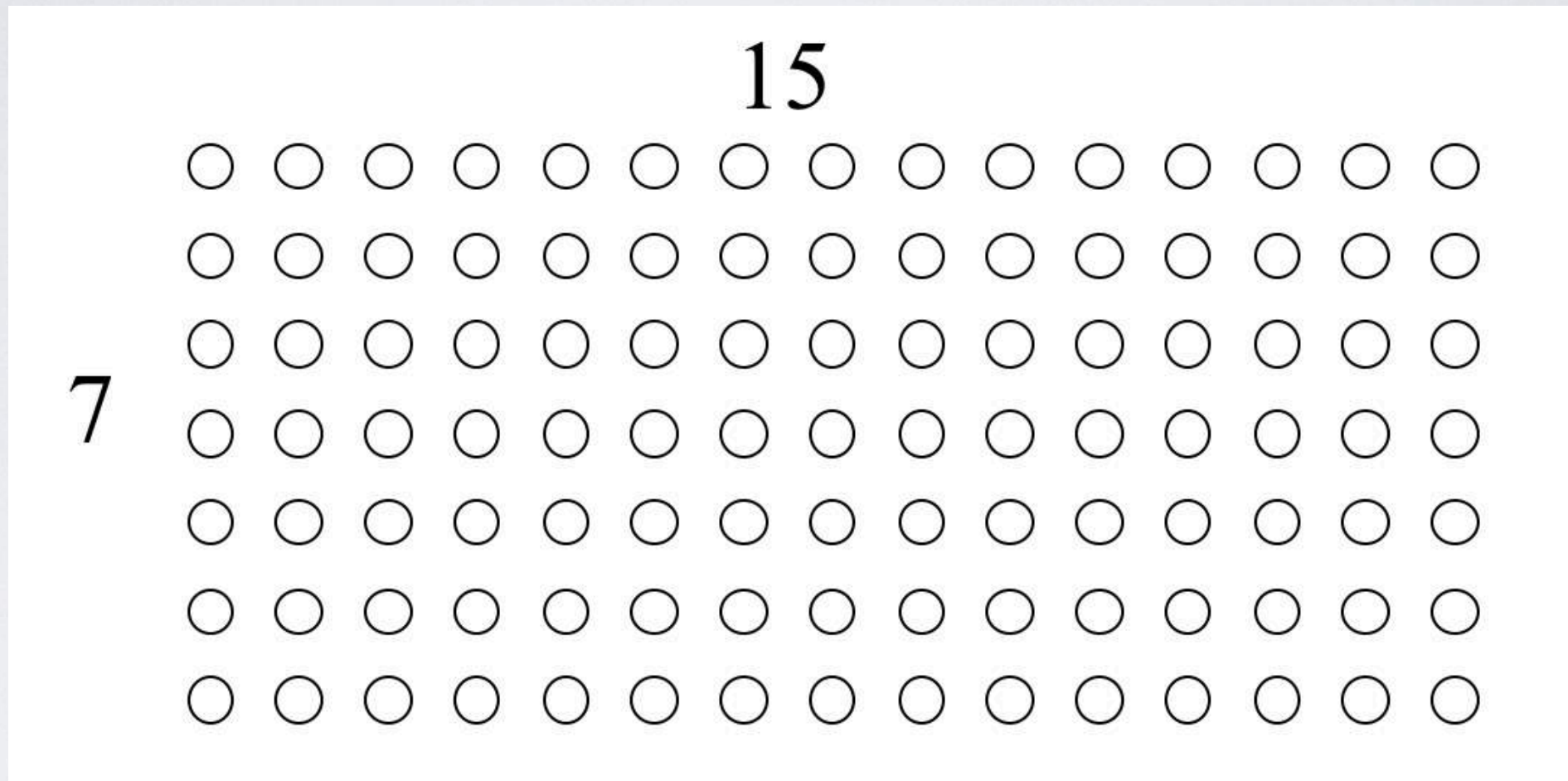
Answer : 14 cm²





Opportunities for low-achievers

REDUCE UNKNOWN TO KNOWN



Grade 3: Multiplication

EXHAUSTIVE LISTING OF FACTORS

Find **all** the factors of 24.

$\overline{)24}$ 24 (is / is not) divisible by 1.	$\overline{)24}$ 24 (is / is not) divisible by 2.	$\overline{)24}$ 24 (is / is not) divisible by 3.	$\overline{)24}$ 24 (is / is not) divisible by 4.	$\overline{)24}$ 24 (is / is not) divisible by 5.	$\overline{)24}$ 24 (is / is not) divisible by 6.
$\overline{)24}$ 24 (is / is not) divisible by 7.	$\overline{)24}$ 24 (is / is not) divisible by 8.	$\overline{)24}$ 24 (is / is not) divisible by 9.	$\overline{)24}$ 24 (is / is not) divisible by 10.	$\overline{)24}$ 24 (is / is not) divisible by 11.	$\overline{)24}$ 24 (is / is not) divisible by 12.
$\overline{)24}$ 24 (is / is not) divisible by 13.	$\overline{)24}$ 24 (is / is not) divisible by 14.	$\overline{)24}$ 24 (is / is not) divisible by 15.	$\overline{)24}$ 24 (is / is not) divisible by 16.	$\overline{)24}$ 24 (is / is not) divisible by 17.	$\overline{)24}$ 24 (is / is not) divisible by 18.
$\overline{)24}$ 24 (is / is not) divisible by 19.	$\overline{)24}$ 24 (is / is not) divisible by 20.	$\overline{)24}$ 24 (is / is not) divisible by 21.	$\overline{)24}$ 24 (is / is not) divisible by 22.	$\overline{)24}$ 24 (is / is not) divisible by 23.	$\overline{)24}$ 24 (is / is not) divisible by 24.

Therefore, _____ are the factors of 24.

VOLUME OF CUBOID

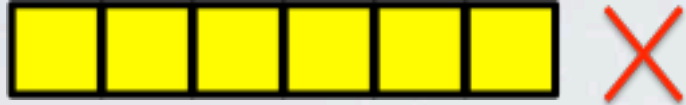
CAKE DISSECTION

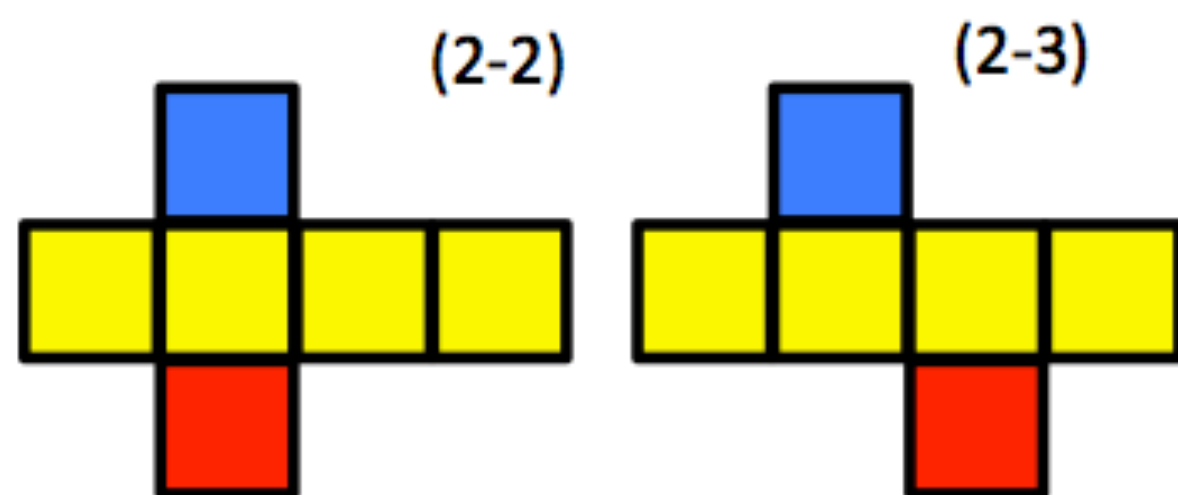
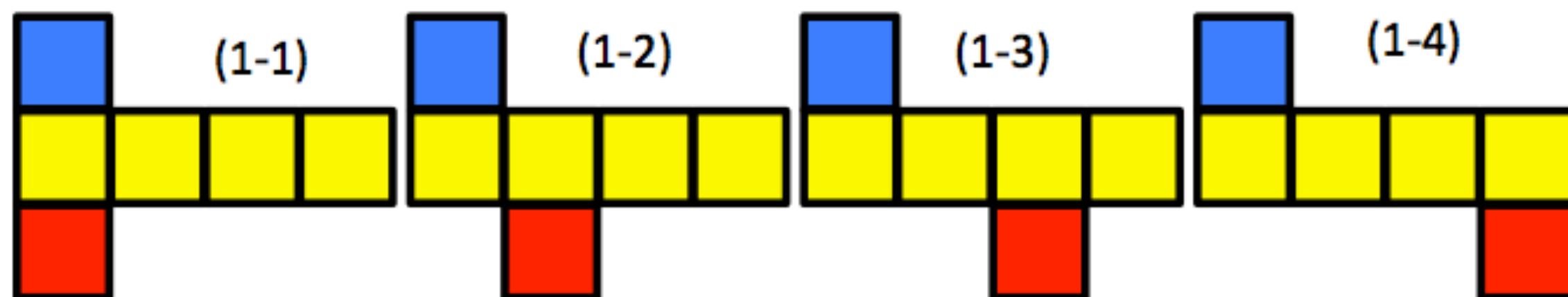


DISCOVERING VOLUME FORMULA

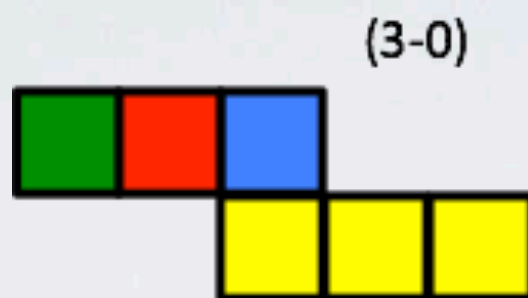


How to find all the possible
nets of cubes?

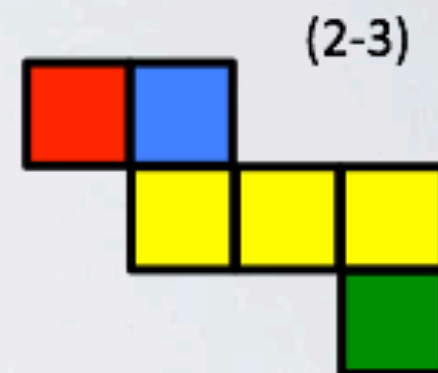
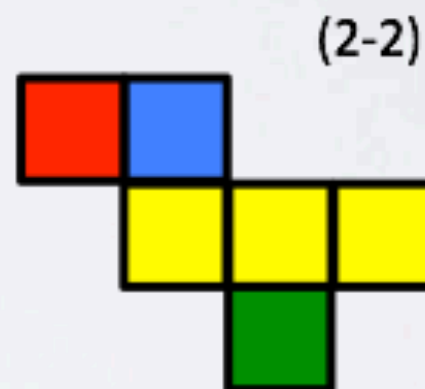
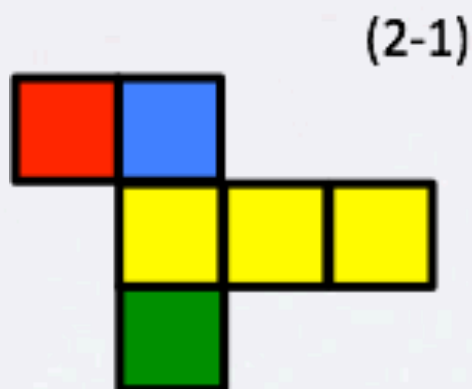




情況一：位於同側：



情況二：位於兩側：





(2-2)

BRAINSTORM

- Think about your lesson plans for next month. Pick the lesson you see as important and design a problem-based task for your students.
- Any task or activity which students have no prescribed rules or memorized procedures that they can use to solve it
- Need not be complex or elaborate

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- 馮振業、葉嘉慧 (2004，6月) 。數學化教學：空間觀念的培養。載鄧幹明、黃家樂、李文生、莫雅慈 (編) 。《香港數學教育會議 - 2004論文集》 (頁89-96) 。香港大學教育學院。