Foundational Inquiries:

AMANDA BEAN'S AMAZING DREAM

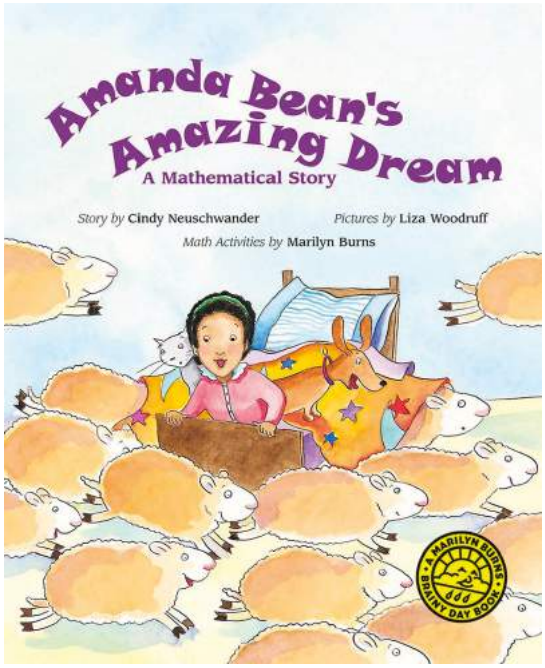
by Emily S.W. Sum

OBJECTIVITIES AND STANDARDS

Understand multiplication as repeated addition of the same quantities; Model multiplicative situations as rows and columns in array models/diagrams; Understand and use the language of multiplication.

You need

- Picture Book: *Amanda Bean's Amazing Dream* by Cindy Neuschwander. (Chinese Version – 阿曼達的瘋狂大夢：乘法的秘密 / 辛蒂.紐斯汪德文；麗莎. 伍卓芙圖；吳瑛譯.)
- Counters or Interlocking cubes
- Cuisenaire rods



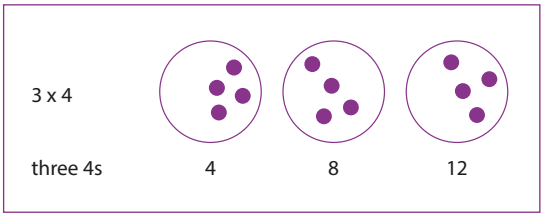
CULTURAL/LANGUAGE CONSIDERATIONS

Mathematical story promotes mathematical languages, starting with the articulation of the problem and ending with the mathematical expression/idea. (See the Cultural and Linguistic Diversity subpage of the website for details.) Teachers need to ensure that mathematical language and vocabulary are secure in a story structure. Sentence structures used in the language of multiplication are challenging for many language learners. For example,

> A gardener is planting seeds in a tray of little pots. A tray holds four rows of five pots. How many seeds can the gardener plant if she puts one seed into each little pot on a tray? (Askew, 2015)

In order to minimize language barrier, teacher should provide hands-on manipulatives and other visual image for multiplication in arrays where objects are arranged in rows and columns. Modeling with the array helps students to understand the underlying mathematical structure.

Also, language learners may be confused by the words used to describe/explain multiplication concepts. For example: times, multiply, multiplied by, lots of, group of. Teacher should limit the number of words used when introducing the concepts. Avoid passive construction. For example, 3 times 4 may be easier to understand than 4 multiplied by 3. 3 groups of 4, 4 taken 3 times is 3 × 4. Reword the sentences if necessary.



DIAGNOSTIC ASSESSMENT

Use the test to diagnose your students' understanding of multiplication and division before you start teaching the lessons. Read the questions out loud (twice) to the class and then give them time to work on the problem before moving on to the next question. The test will give you a sense of students' prior knowledge on multiplication and division.

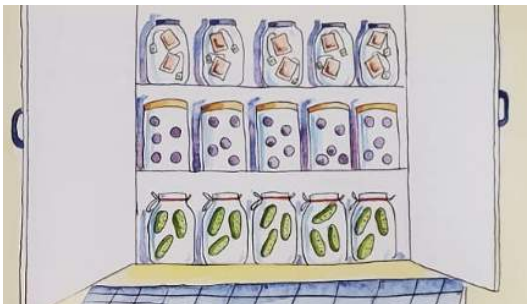


Fig. Amanda Bean's Amazing Dream

1. Amanda has 5 jars of teabags. There are 2 teabags in each jar. How many teabags does Amanda have? *(equal groups, multiplication)*
2. Amanda has 5 jars of candies. There are 5 candies in each jar. How many candies does Amanda have? *(equal groups, multiplication)*
3. Amanda has 15 gherkins/cucumbers. She puts the gherkins into the jars. Each jar can hold 3 gherkins. How many jars does Amanda need? *(equal groups, division)*
4. Look at the picture. How many jars are there altogether? *(arrays, multiplication)*
5. Can you suggest any other arrangement? *(arrays, multiplication and division)*

Mark the test:

0 Incorrect answer

1 Correct operation/drawings/reasoning but incorrect answer OR incorrect operation/drawings/reasoning but correct answer

2 Correct (OR no) operation/drawings/ reasoning and correct answer

And code the strategies used.

A [Repeated] Addition

B Multiplication

C Drawings

D Reasoning or explanation in words

E Division

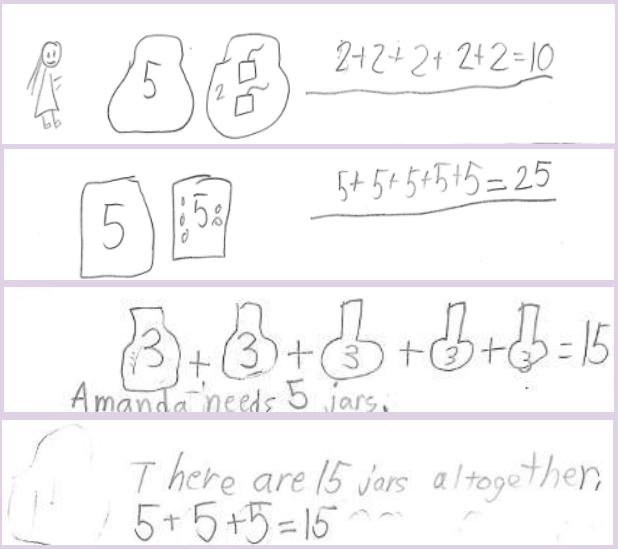
F [Repeated] Subtraction

G No strategy

For example, 0B is coded for incorrect answer as multiplication strategy was used.

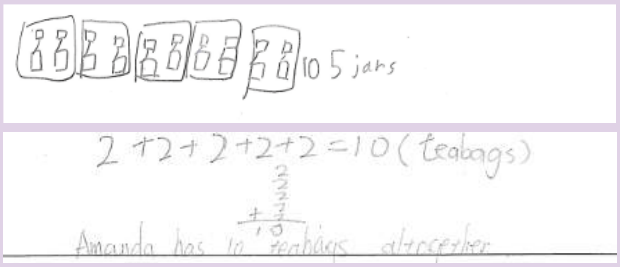
STUDENTS CONSTRUCT MATHEMATICAL UNDERSTANDING

This mixed-race student used repeated addition strategy to solve all problems.



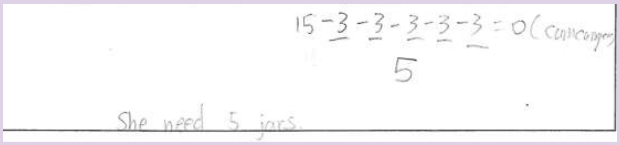
Other students' examples:

Additive thinking

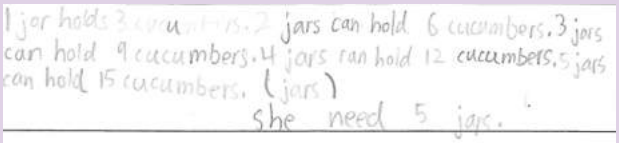


Multiplicative thinking

Repeated subtraction

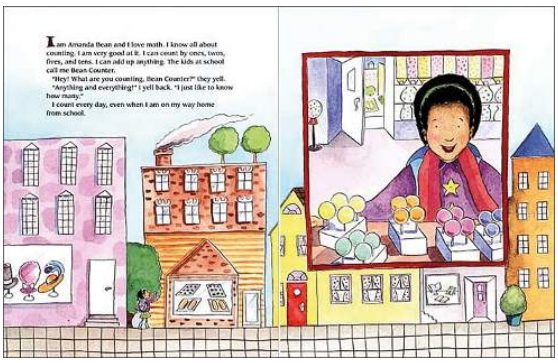


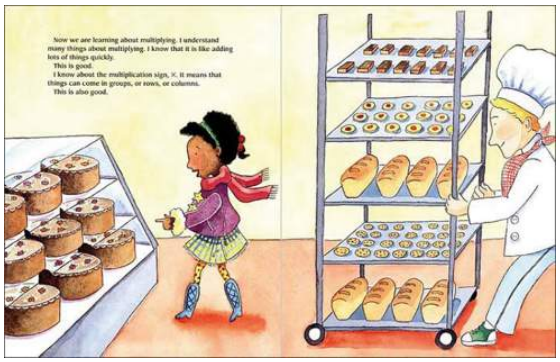
Reasoning



DIRECTION

Read the story. The context lends itself to students modelling multiplicative situations as arrays with the aid of objects or by means of drawing.





Activate students' prior knowledge with warm-up activities involving counting by ones, twos, fives and tens.

Play with this in different ways, for example

Take in turns for teacher/students to count, saying alternate numbers in sequence.

Repeat starting at 20 …

Questions for Students

Trays of bread and cookies had just arrived, and the loaves of bread were laid out on the tray...

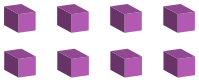
If there were 4 loaves of bread on the tray,

1. How many loaves of bread were there altogether?
2. How many stripes are there on all the loaves of bread?

Ask students to represent the arrangement (2-by-4 array) using counters/cubes.

1. How many rows?
2. How many cubes in each row?
3. How many cubes in the array?

The array can be annotated accordingly.



Students may say that they added four and four. Take their explanations which then lead to introducing the multiplication sign. Write the mathematical expression/equation to represent the situation.

Talk about the methods in finding the total.

- Is it better to count in ones or in groups of twos/fours?

Embeded in these arrays is the mathematical idea of the commutative rule, which students develop intuitively and articulate without knowing the name of it.

Read the story. Pause where appropriate to assimilate details in the illustrations.

Talk by posing more problems based on the picture, for example jam tarts, chocolate chip cookies etc. Encourage students to set up array models with counters/cubes or drawing arrays to find the answer using whatever method they like. Ask them to write the mathematical expressions/equations that match the situations.

Invite students to share their solutions with the class. Look out for anyone saying "number of jam tarts in one row/column" and ask them how they figure out the total without counting every tart individually. Discuss efficient ways to find the total number of jam tarts.

Is it better to count groups of 3 or groups of 6? What if there were 7 tarts in each row? What if there were 1 extra row?

Students can count objects in arrays without mastering addition/multiplication.

Questions for Students

1. The pastry chef puts éclairs in rows on a tray. Each row has 7 éclairs. He puts out 2 rows. How many éclairs does he put out?
2. The pastry chef puts jam tarts in rows on a tray. Each row has 6 jam tarts. He puts out 3 rows. How many jam tarts does he put out?
3. The pastry chef puts chocolate chip cookies in rows on a tray. Each row has 7 chocolate chip cookies. He puts out 4 rows. How many jam tarts does he put out?

Compare and discuss your answers in Q1 and Q3.

As students describe/construct the arrays of different multiplicative situations (groups of lollipops, wheels on the bicycles, balls of yarn, pairs of knitting needles). Later in the story talk about the patterns of twos, threes, fours, and fives in developing students' multiplicative reasoning.

PROBLEM POSING

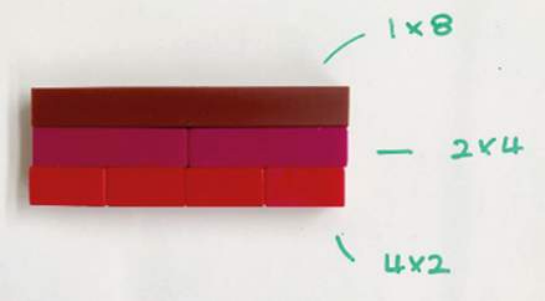
Posing questions/problems presents important thinking challenges for students. And by playing with the story plots teacher can pose new problems based on the story. For example, ask students for ideas on how they could count the popcorn in a bowl. Do a think-pair-share and then ask for pairs to share with the class. Students may suggest organizing/grouping the popcorn by twos, fives or other ways.

CREATING PROBLEMS

Students can "make up a story problem" which is similar to one of these problems. This involves thinking on a higher level that is beyond applying what one already knows. Unlike + means "add" and – means "take away", multiplication × "times" is one of the most difficult operations for students to illustrate. "Make up story" develops students' understanding and builds mathematical vocabulary.

VARIATION

The development of the mathematical meaning can be assisted by manipulative or other concrete representations as students visualize mathematical ideas through the story and experience creative mathematical thinking. For instance, ask students to find pairs of numbers whose product is 8 using C-rods.



ASSESSMENT

1. Can students describe the multiplication situations using array model/diagram?
2. Can students write a mathematical expression or equation that matches the situation?
3. Can students use additive/multiplicative strategies for finding the total number of objects?

Questions for Students

How many different pairs can you find?

A/One group of 8 = 1 × 8

Two groups of 4 = 2 × 4

Four groups of 2 = 4 × 2

Repeat this with other numbers.

The array model helps student to develop the relationship between numbers.

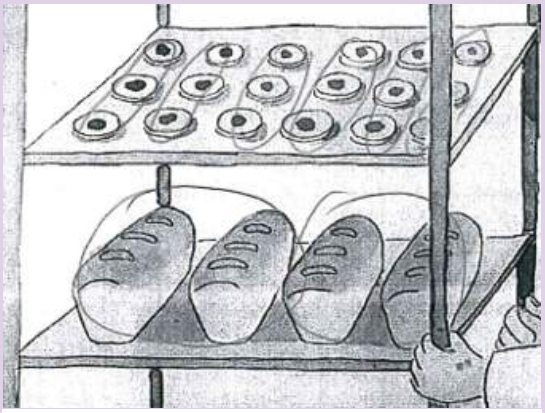
Another model that complements the use of arrays is T-table*. For example,

Tray	loaves of bread
1	4
2	8

and so on⋯

*T-table makes the relationship of the two quantities explicit and helps students deepen their understanding of multiplication.

STUDENT WORK



(c) How did you arrange the ?

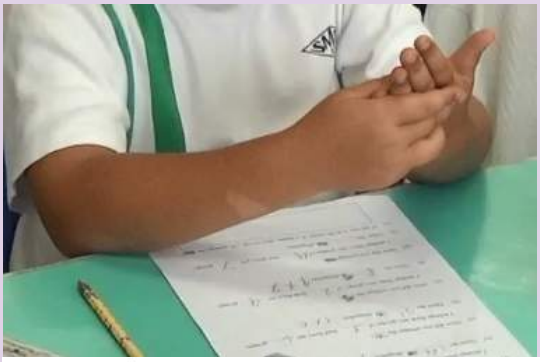
I arrange them into groups of 3. And there are 6 groups.

(d) There are 18 altogether.

(e) How did you arrange the ?

I arrange them into groups of 2. And there are 2 groups.

(f) There are 4 altogether.



(a) How did you arrange the ?

I arrange them into groups of 2. And there are 7 groups.

(b) There are 14 altogether. 2 X 7

(c) How did you arrange the ?

I arrange them into groups of 3. And there are 6 groups.

(d) There are 21 altogether. 3 X 6

(e) How did you arrange the ?

I arrange them into groups of 2. And there are 4 groups.

(f) There are 8 altogether. 4 X 2

While the story is being told, students might imagine themselves to be Amanda by projecting themselves into her person and experiences. Amanda's questions might become students' questions, and her mathematical actions might be viewed as their actions. Students would construct for themselves the storyline, their own personal mathematical meanings, their views of mathematics embodying a set of procedures needed to solve problems. The story allows students to see the mathematics they are learning in the context of specific adventures, to do mathematics by grouping things, and to think by using cubes.

In this case, students were asked to find the correct number of cakes by equal grouping, without being told how the calculation is done.

REFERENCE

Askew, M. (2015) *A practical guide to transforming primary mathematics: Activities and tasks that really work*. Taylor and Francis, Florence.