

LEARNING DIVISION OF FRACTIONS THROUGH LANGUAGE

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Background



P5Gp1 (able class)



27 students including 5 Non-Chinese students



Lessons were conducted on Zoom

Why do we choose division of fractions?

- The explanation in the textbook is too brief
- Beyond Algorithm
 - *Students always focus on the routine calculation rather than the conceptual understanding of division of fractions*
 - Partition
 - Quotition
- Some research evidence suggests that early emphasis on procedures over concepts can actually impede students' fractions understanding in the long term (Brown & Quinn, 2007)

Teaching sequence

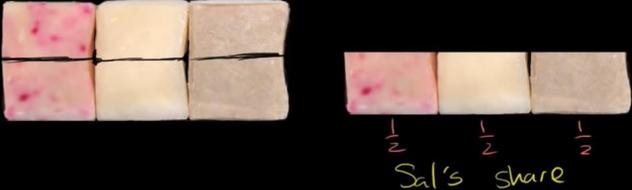
- Divide a fraction by a whole number
- Divide a whole number by a fraction
- Divide a fraction by a fraction
- Story-writing by students

Divide a fraction by a whole number

- Introduce the concept of division by diagram
 - *Turn the division to multiplication (by taking the reciprocal of the divisor)*
- Make sure students understand “Why” but not just “How”
- And which numbers (dividend or divisor) to take reciprocal

 Creating a fraction through division of whole numbers | Fractions | Pre-Algebra | Khan Academy

$3 \div 2 = 3 \cdot \frac{1}{2}$



$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
Sal's share

3 divided by 2 is equal to 3 times $\frac{1}{2}$.



$$3 \div 2 = 3 \cdot \frac{1}{2}$$



$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

Sal's share

3 divided by 2 is equal to 3 times 1/2.

Divide a whole number by a fraction

- Use language to provoke their thinking together with diagram to present their answer
- Language
 - *Unit fractions, e.g. One-half, one-third, one-fifth, ...*
 - *E.g. “**How many $1/4$ (one-fourth) in $1/2$ (a half)?**”*
- Pre-lesson task
 - *Questions are specially designed*
 - *Use diagram to show students’ thinking*

Consider the difference in a student hearing

1 OVER 4 times 2

Saying 1 over 4 communicates to students that the fraction is two whole numbers.

1-fourth of 2

Saying fractions using “*ths*” connects to students that the fraction represents a single value.

This also develop a better understanding of division of fractions,

e.g. **$4 \div 1/8$**

How many eighths are in 4?

Using precise language that focuses on the meaning of fractions and on equivalence, can play an important role in helping students understand fractions (including operations and representations and use them in efficient and accurate ways. (Bay-Williams, 2013)

1. How many one-fourth in a whole? (Draw a picture/diagram to show your thinking)



2. How many two-thirds are there in two? (Draw a picture/diagram to show your thinking)



Pre-lesson Worksheet

- Focus on language
- E.g. One-fourth, whole, two-thirds, two

Students' work

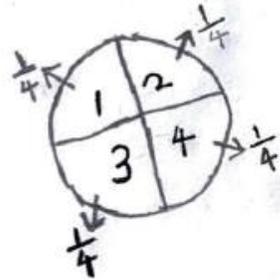
1. How many one-fourth in a whole? (Draw a picture/diagram to show your thinking)

$$1 \div \frac{1}{4}$$

$$= 1 \times 4$$

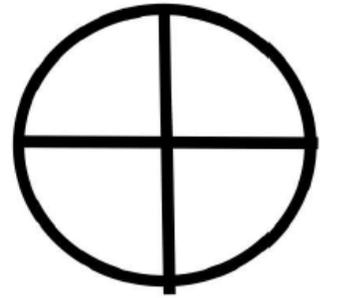
$$= 4 \text{ (one-fourths)}$$

There are 4 one-fourths in a whole.



1. How many one-fourth in a whole? (Draw a picture/diagram to show your thinking)

There are 4 one-fourths in a whole.



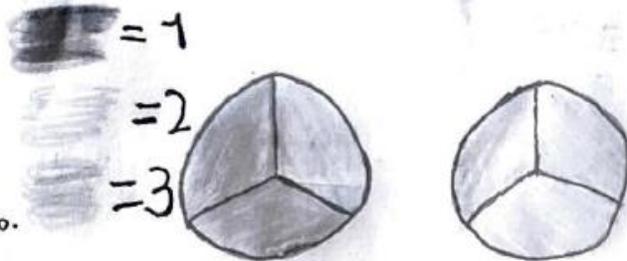
2. How many two-thirds are there in two? (Draw a picture/diagram to show your thinking)

$$2 \div \frac{2}{3}$$

$$= 2 \times \frac{3}{2}$$

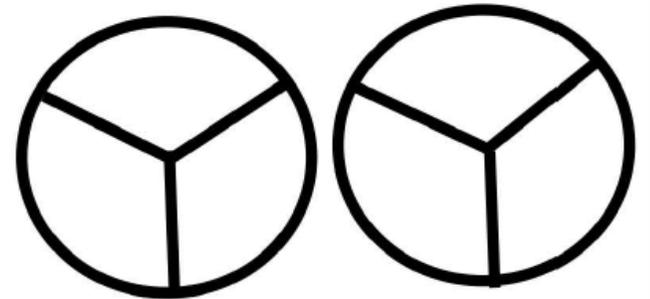
$$= 3 \text{ (two-thirds)}$$

There are 3 two-thirds in two.



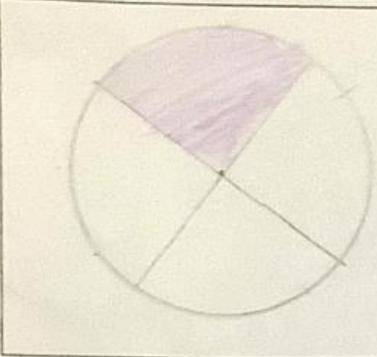
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There are 3 two-thirds in two.



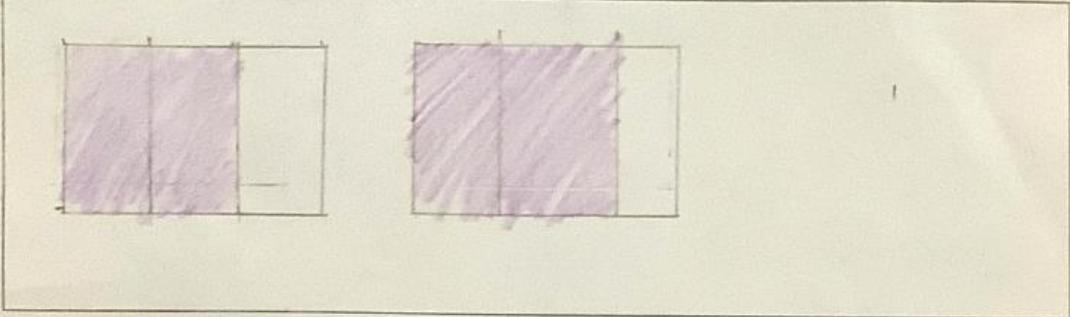
Students' work

1. How many one-fourth in a whole? (Draw a picture/diagram to show your thinking)



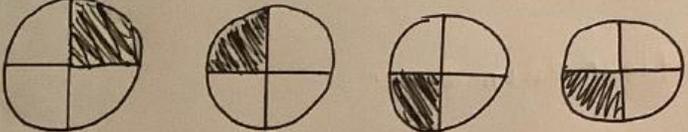
Able to show one-fourth

2. How many two-thirds are there in two? (Draw a picture/diagram to show your thinking)



Can show “two-third in a whole” but may not be able to count how many there

1. How many one-fourth in a whole? (Draw a picture/diagram to show your thinking)

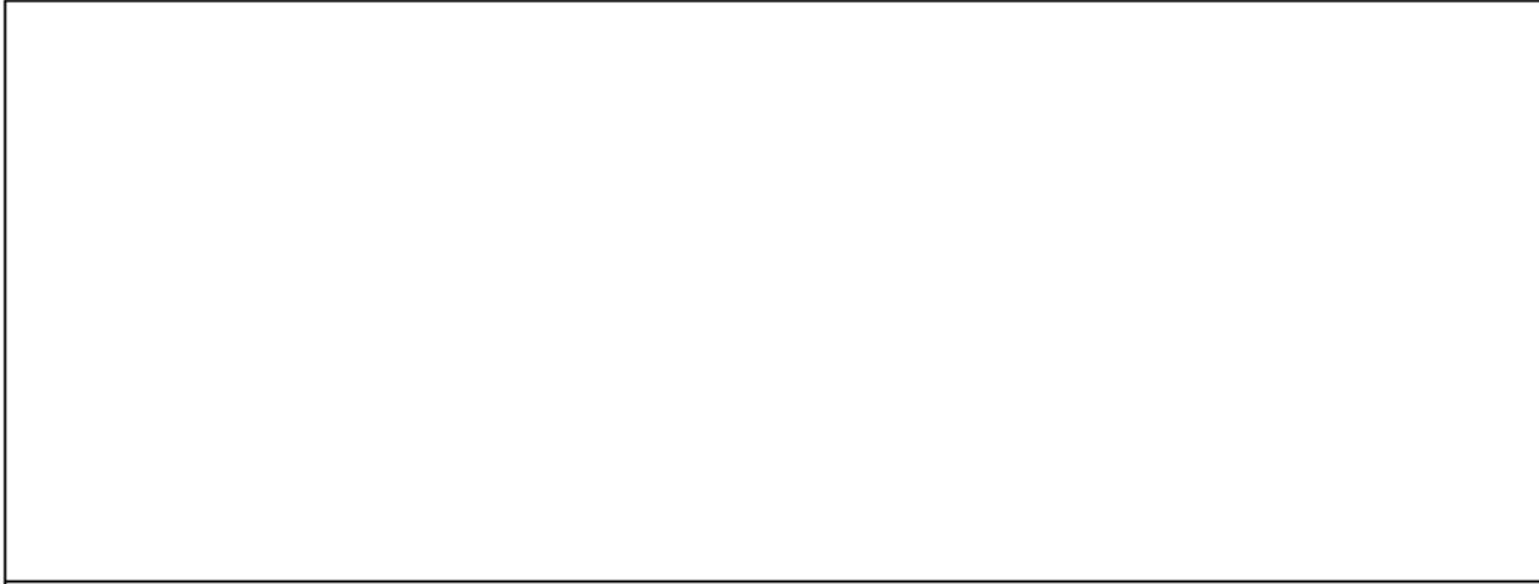

$$4 \times \frac{1}{4}$$
$$= 4$$

2. How many two-thirds are there in two? (Draw a picture/diagram to show your thinking)


$$3 \times \frac{2}{3}$$
$$= 2$$

Misunderstand the whole

4. Peter has $\frac{5}{6}$ kg of flour. He uses an equal amount of flour for baking every day. After 5 days, he uses all his flour. How much flour does he use each day? (Draw a picture/diagram to show your thinking)



5. Peter eats $\frac{1}{10}$ kg of cookies every day. How many days will he finish a bag of cookies that is $\frac{1}{2}$ kg?
(Draw a picture/diagram to show your thinking)



Divide a fraction by a fraction

- Pre-lesson Worksheet
 - Q4: *Partition*
 - Q5 *Quotition*

Divide a fraction by a fraction

■ Partition

4. Peter has $\frac{5}{6}$ kg of flour. He uses an equal amount of flour for baking every day. After 5 days, he uses all his flour. How much flour does he use each day? (Draw a picture/diagram to show your thinking)

He uses $\frac{1}{6}$ kg of flour each day.

4. Peter has $\frac{5}{6}$ kg of flour. He uses an equal amount of flour for baking every day. After 5 days, he uses all his flour. How much flour does he use each day? (Draw a picture/diagram to show your thinking)

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$\frac{5}{6} \div 5$
 $= \frac{5}{6} \times \frac{1}{5}$
 $= \frac{1}{6} \text{ (kg)}$

Day 1 Day 2 Day 3 Day 4 Day 5
 $\frac{1}{6} \text{ kg}$ $\frac{1}{6} \text{ kg}$ $\frac{1}{6} \text{ kg}$ $\frac{1}{6} \text{ kg}$ $\frac{1}{6} \text{ kg}$

He uses $\frac{1}{6}$ kg of flour every day.

Divide a fraction by a fraction

■ Quotition

5. Peter eats $\frac{1}{10}$ kg of cookies every day. How many days will he finish a bag of cookies that is $\frac{1}{2}$ kg?

(Draw a picture/diagram to show your thinking)

He will take 5 days to finish a bag of cookies.

Everyday $\frac{1}{10}$ kg + $\frac{1}{10}$ kg + $\frac{1}{10}$ kg + $\frac{1}{10}$ kg + $\frac{1}{10}$ kg

Students were able to expand the fractions of $\frac{1}{2}$ to $\frac{5}{10}$

5. Peter eats $\frac{1}{10}$ kg of cookies every day. How many days will he finish a bag of cookies that is $\frac{1}{2}$ kg?

(Draw a picture/diagram to show your thinking)

Peter eats $\frac{1}{10}$ P/day

day 1 $\frac{5}{10}$ \swarrow $\frac{1}{10}$

day 2 $\frac{4}{10}$ \swarrow $\frac{1}{10}$

day 3 $\frac{3}{10}$ \swarrow $\frac{1}{10}$

day 4 $\frac{2}{10}$ \swarrow $\frac{1}{10}$

day 5 $\frac{1}{10}$ \swarrow $\frac{1}{10}$

day 6 0 \swarrow $\frac{1}{10}$

5. Peter eats $\frac{1}{10}$ kg of cookies every day. How many days will he finish a bag of cookies that is $\frac{1}{2}$ kg?

(Draw a picture/diagram to show your thinking)

He will finish the bag in 5 days.

$\frac{1}{2} = \frac{5}{10}$

A $\frac{1}{2}$ kg bag of cookies.

1/10 kg- day 1

2/10kg- day 2

3/10kg- day 3

4/10 kg-day 4

5/10kg-day 5

5. Peter eats $\frac{1}{10}$ kg of cookies every day. How many days will he finish a bag of cookies that is $\frac{1}{2}$ kg?

(Draw a picture/diagram to show your thinking)

$\frac{1}{2} \div \frac{1}{10}$

$= \frac{1}{2} \times 10$

$= 5$ (days)

He finish a bag of cookies that is $\frac{1}{2}$ kg in 5 days.

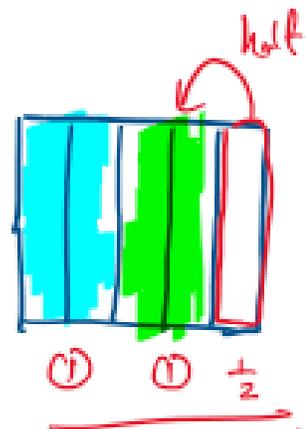
After the Pre-lesson worksheet.....

- Some students do not have a solid understanding of the concepts
- They understood the language (unit fractions) we used in the worksheet
- But they rather perform calculation instead of drawing diagram to show two underlying concepts
 - *Partition*
 - *Quotition*

FURTHER CONSOLIDATION

The remaining white part is **HALF** of the green part

2. How many two-fifth are there in one?



half of previous

① ① $\frac{1}{2}$

$$2\frac{1}{2} \text{ two fifths in one}$$

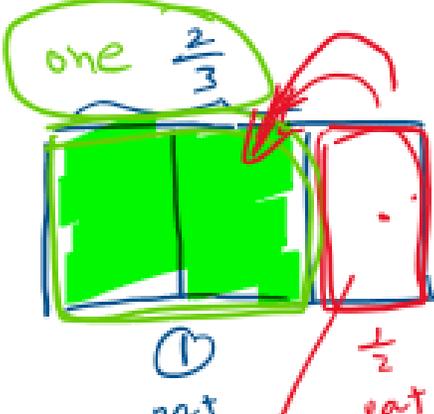
$$1 \div \frac{2}{5}$$

$$= 1 \times \frac{5}{2}$$

$$= \frac{5}{2}$$

$$= 2\frac{1}{2}$$

3. How many two third are there in one?



one $\frac{2}{3}$

$\frac{1}{2}$ of the green part

① part $\frac{1}{2}$ part

ans: $1\frac{1}{2}$

$$1 \div \frac{2}{3}$$

$$= 1 \times \frac{3}{2}$$

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$

?

$\frac{1}{3}$ of one

Zoom Teaching

- An example in the textbook which is the same as Q2 of Pre-lesson worksheet

There are 12 pieces of pizzas.

2 Divide 2 kg of cookies into bags of $\frac{2}{3}$ kg. How many bags of cookies are there?

Handwritten notes: $\frac{2}{3}$ or 1 kg

Each portion has $\frac{1}{3}$ kg of cookies.

Each bag has $\frac{2}{3}$ kg of cookies. (Each bag has 2 portions of cookies.)

$2 \times 3 = 6$
There are 6 portions.

$6 \times \frac{1}{2} = 3$
There are 3 bags of cookies.

Handwritten note: 2 kg

$2 \div \frac{2}{3}$
 $= 2 \times 3 \times \frac{1}{2}$
 $= 2 \times \frac{3}{2}$
 $= 3$

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

So, there are 3 bags of cookies.

3 How many $\frac{3}{8}$ are there in $\frac{3}{4}$?

Handwritten note: Answer

$\frac{3}{4} \div \frac{3}{8}$
 $= \frac{3}{4} \times \frac{8}{3}$

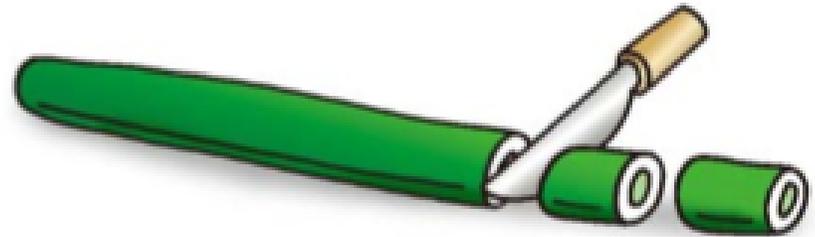
Reciprocal of $\frac{3}{8}$.



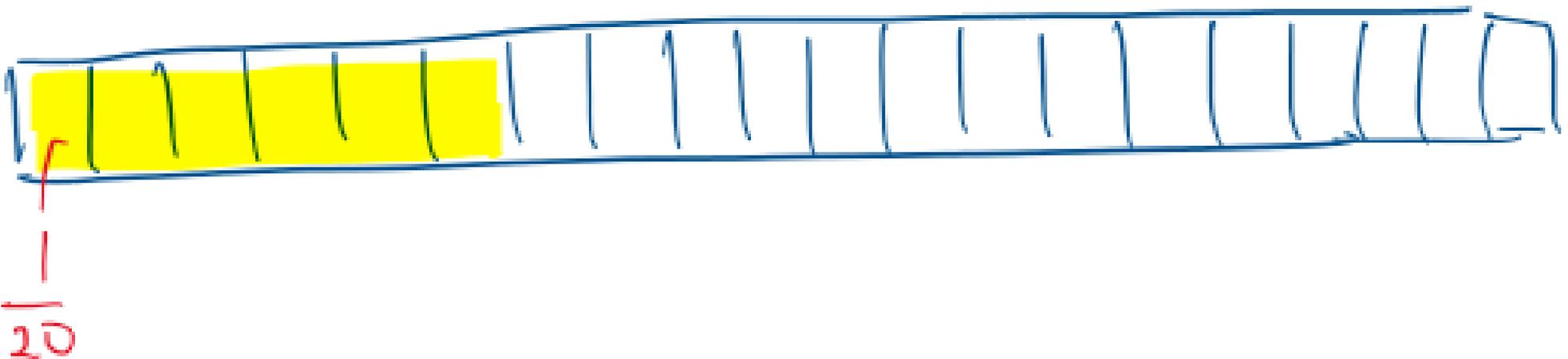
A roll of sushi is $\frac{3}{10}$ m long. It is cut into slices of $\frac{1}{20}$ m. How many slices of sushi are there?

$$\begin{aligned} & \frac{3}{10} \div \frac{1}{20} \\ &= \frac{3}{10} \times \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

$$\frac{6}{20}$$



There are _____ slices of sushi.



Using “Number line” to represent the answer

2. Mum eats $\frac{1}{8}$ kg of cheese in $\frac{1}{4}$ month. How many kilograms of cheese does she eat in one month?

(Use a picture / diagram to represent your answer).

3. Dad drives 10 km in $\frac{1}{3}$ hours. How many kilometers does he drive in one hour? (Use a picture / diagram

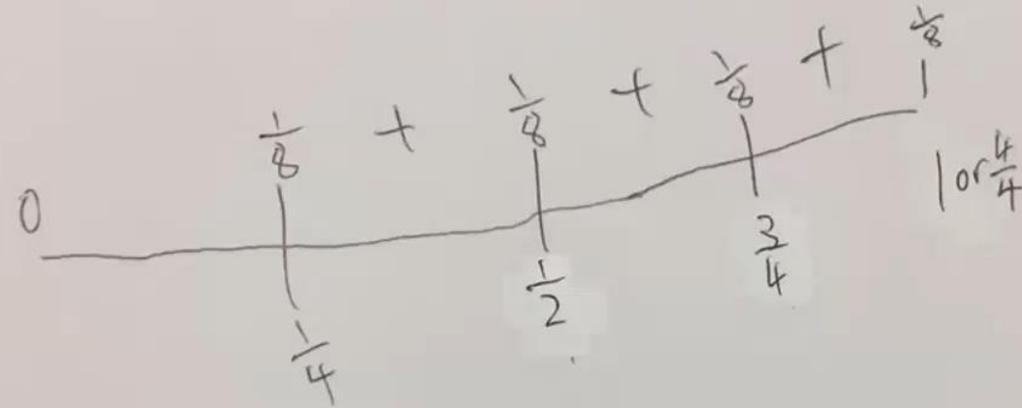
to represent your answer).

4. Tom walks $1\frac{1}{2}$ km in $\frac{3}{8}$ hours, how much time does it take for him to walk 1 km? (Use a picture /

diagram to represent your answer).

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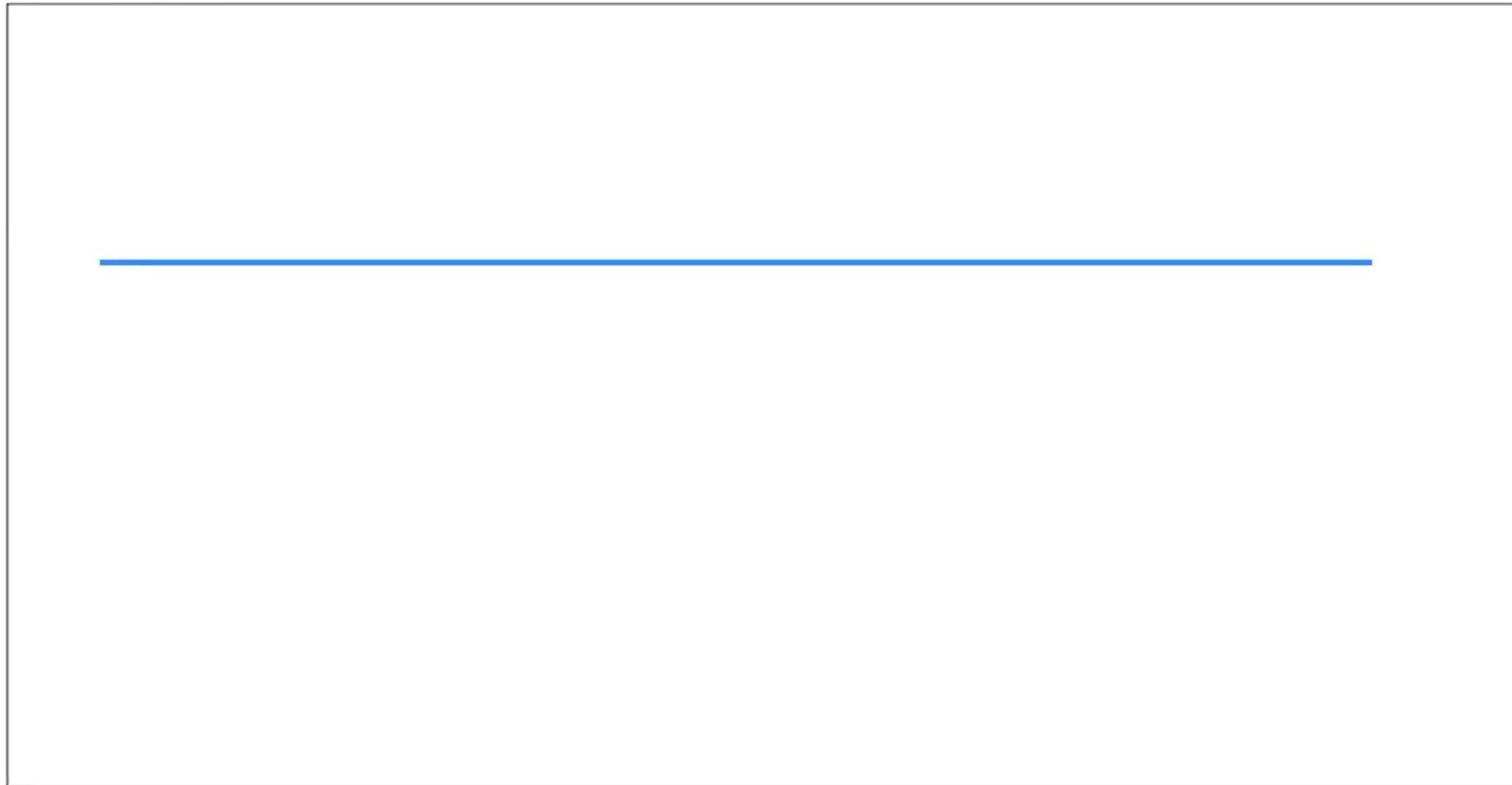
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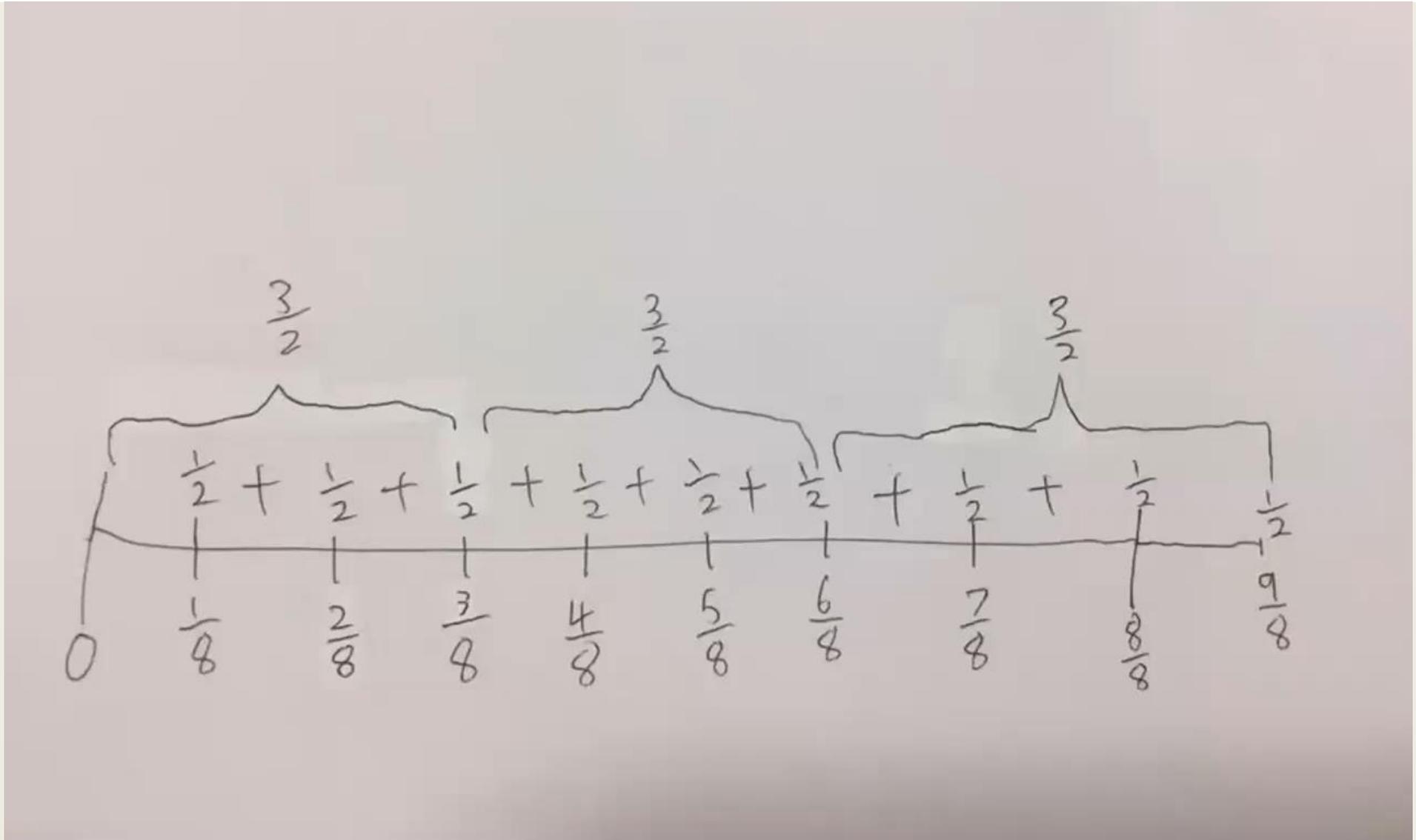
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<

3. Dad drives 10 km in $\frac{1}{3}$ hour. How many kilometers does he drive in one hour? (Use a picture / diagram to represent your answer).



4. Tom walks $1\frac{1}{2}$ km in $\frac{3}{8}$ hours, how much time does it take for him to walk 1 km? (Use a picture / diagram to represent your answer).



Story writing by students

■ Instruction

- *Work in Group of 3 in the lesson*
- *Write a story/ word problem on division of fraction.*
- *You are going to check the other groups' story. All stories will be used in the class tomorrow (7 Apr)*

■ Example

- *Tom walks $1\frac{1}{2}$ km in $\frac{3}{8}$ hours, how much time does it take for him to walk 1 km?*
(Use a picture / diagram to represent your answer).

Story writing by students

Problems in their stories – not about division

Chloe Chang, Alice, Cameron

4. One day a boy went to a buffet restaurant, there are 2 L of juice and 2 kg of meat, he got $\frac{1}{5}$ of the juice and $\frac{1}{6}$ of the meat, how much meat and juice is left?

Edgar, Mario, Bosco

5. 3 people go to the beach by taxi. The taxi's speed is $50\frac{5}{8}$ km per hour. The taxi's speed then decreases by $\frac{7}{8}$ times of the original speed. What is the taxi's speed now?

Aditya, chloe li, lester

6. Tom's weight is $\frac{1}{5}$ of his dad. His dad's weight is $1\frac{1}{2}$ of his mum. If Tom's mum is 60 kg, how heavy is tom?

Tom's Dad

With some modification:

Aditya

6. Tom's weight is $\frac{5}{6}$ of his mum. His dad's weight is $1\frac{1}{2}$ of his mum. If Tom's dad weighs 81 kg, how heavy is Tom?

his mum's weight:

Aditya, chloe li, Lester

6. Tom's weight is $\frac{1}{5}$ of his dad. His dad's weight is $1\frac{1}{2}$ of his mum. If Tom's mum is 60 kg, how heavy is tom?





Aditya , chloe li. Lester

6. Tom's weight is $\frac{1}{5}$ of his dad. His dad's weight is $1\frac{1}{2}$ of his mum. If Tom's mum is 60 kg, how heavy is tom?

$$60 \times \underbrace{1\frac{1}{2}}_{\text{Dad}} \times \underbrace{\frac{1}{5}}_{\text{Tom}}$$

Our Experiences

- Strategies

1. *Language used*

- Using “one-third” rather than “one over three”

2. *Multiple representation – using rectangle, number line or other pictures*

3. *Focus on Unit fractions*

- Teaching on Zoom

- *Hard to monitor every student’s learning and check their work*
- *Can only ask some students to show their work on screen*

Our Experiences

- Collaboration with other P5 teachers
 - *Further discussion about teachers in compromising the direction of teaching is needed*
 - *Face-to-face meeting is recommended*
- Difficulties
 - *The 1st worksheet*
 - New kind of teaching methods which need time to brainstorm
 - *Language*
 - Get used to say e.g., “one over four” instead of “one-fourth”

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THANK YOU