Using Contextual Problems in Developing Students' Meanings for Addition and Subtraction

by Emily S.W. Sum

LEARNING OUTCOMES

- Solve one-step word problems that involve addition and subtraction, using manipulatives/pictorial representations

APPROACHES IN SOLVING WORD PROBLEMS

1. Start with contexts that students can relate to, and make sense of the context through actions, visual aids and other hands-on materials. Manipulatives and multiple graphic representations can be helpful for language learners by minimising language barriers, and remember not to be mistaken for learning problem/deficit (Wiest, 2008). Moving between representations allows students to develop vocabulary, and link written/spoken language to symbols, pictures, and other concrete objects (Cady, Hodges, & Brown, 2010).
2. Ask students to draw/complete a diagram that matches the sematic structure of the problem. This helps students become aware of the semantic relation between the quantities, and use that knowledge for understanding the problem situation.
3. Represent the situation with graphic representations, and the related number sentence and symbols, before preforming computation. Give students ample opportunities to listen, speak and write mathematically (Anstrom, 1999). Writing down the key vocabulary items on the board while saying them can help students who may not recognise the spoken words as the same words they are familiar with in written form (Weist, 2008). Also, try to limit the use of nonessential or confusing vocabulary.

CULTURAL/LANGUAGE CONSIDERATIONS

Word problems may be difficult for cultural and linguistically diverse learners not just because of the language but also the sentence structure of the problem that is usually different from conversational language. Besides, familiar vocabulary is often used in other ways and this can be challenging for language learners. For example, "how many are left?". Left asks about a reminder whereas in "turn left", it gives a direction. This can be confusing when the directional meaning of the word is most commonly used in everyday language. There are many other words that take on different meanings in mathematics from everyday usage, e.g. odd, even, sum, product, mean. Bresser (2003, p.294) points out that "[Students] also may be confused if the same mathematical operation can be signaled with a variety of mathematics terms, such as add, and, plus, sum, and combine". Thus, teachers should teach explicitly the meanings of words in different mathematical contexts.

Research shows "word problems that can be solved by the same arithmetic operation but differ with respect to their underlying semantic structure have very different degrees of difficultly" (Corte & Verschaffel, 1991, p.119). Academic language, the language specific to a content area such as mathematics (Cummins, 1994) is harder to learn because it is not used in student's everyday life. Students usually struggle with the language required to interpret problems, and therefore, teachers should be mindful of whether it is language that is preventing students from demonstrating their mathematical knowledge and understanding. Problems should be presented in multiple ways/ representations, and allow students to show possible alternative methods for solving problems. Reword problems verbally so that the sematic relations are made more explicit without affecting the underlying semantic structure.

ADDITION AND SUBTRACTION PROBLEM STRUCTURES

Presenting a variety of problem types supports students in developing a schema to structure their thinking/reasoning, and understanding the meaning of operations. Make sure to pose problems that include all categories with unknown quantity in different locations so students gain experience thinking/reasoning about and solving a variety of situations. Addition and subtraction word problems can be classified into three semantic categories (Riley et. al, 1983), and they are:

A. Change

B. Combine (Part-part-whole)

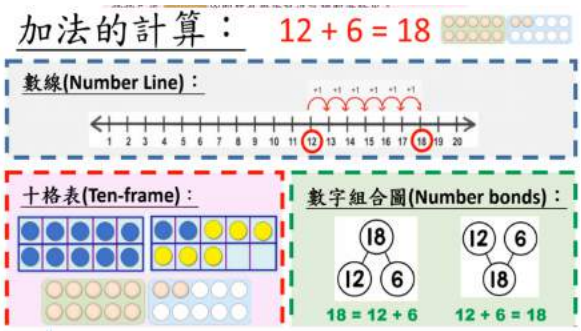
C. Compare

A. Change Situations refer to dynamic situations in which some events change the value of a quantity, e.g. adding to or taking away from a set, which involves a physical action.

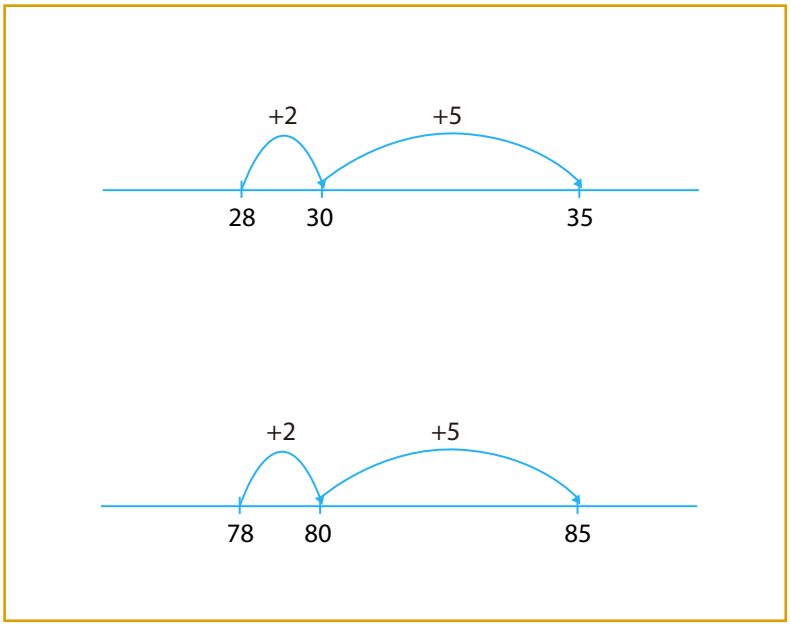
Depending on which of the three quantities (start amount, change amount, and resulting amount), a different problem type results as shown in Table 1 (adapted from Corte & Verschaffel, 1991).

Type	Example	Direction	Unknown
1	Lee had 8 momos, then Sita gave him 7 more momos; how many momos does Lee have now?	Increase	Result set
2	Lee had 15 momos; then he gave 7 momos to Sita; how many momos does Lee has now?	Decrease	Result set
3	Lee had 7 momos; then Lee gave him some more momos; now Lee has 15 momos; how many momos did Lee give him?	Increase	Change set
4	Lee had 15 momos; then he gave some momos to Sita; now Lee has 8 momos; how many momos did he give Lee?	Decrease	Change set
5	Lee had some momos; then Lee gave him 7 more; now Lee has 15 momos; how many momos did Lee have in the beginning?	Increase	Start set
6	Lee had some momos; then he gave 7 momos to Sita; now Lee has 8; how any momos did Lee have in the beginning?	Decrease	Start set

Note that the difficulty between variants of change problems may due to the tenses of the verbs in the problem text.



(Online teaching: Using multiple representations to make sense within the context of the problem situation.)

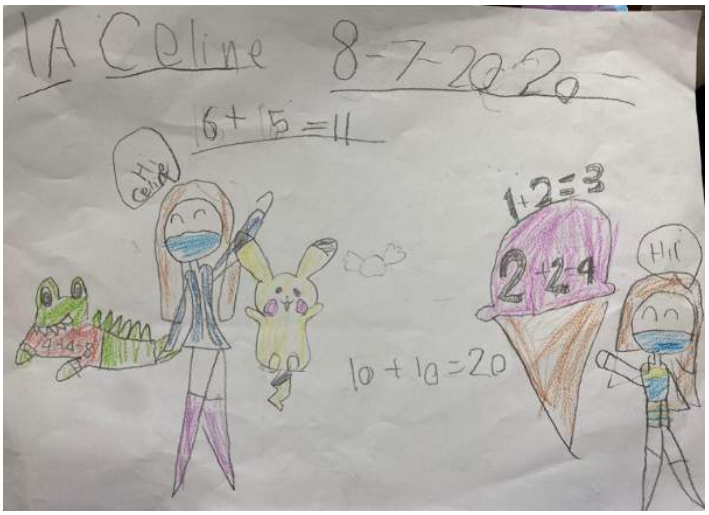


(Online teaching: Using multiple representations to make sense within the context of the problem situation.)

PROBLEM-SOLVING SUPPORT FOR LANGUAGE LEARNERS

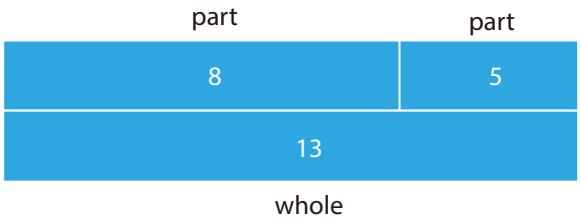
In this lesson, the teacher designs contextual problems that connect closely to students' lives. She uses techniques to acknowledge and validate culturally and linguistically diverse students such as personalising word problems in student names, which is much appreciated in a multicultural classroom. Based on the one single context, the teacher explores problems with great depth and breadth in one class session, which is much preferred over solving numerous standard textbook word problems in different contexts. By doing this, students do not have to reorient their thinking or gain new information for each new problem. The contextual problems are designed to be comprehensible for students without lowering/simplifying the mathematical content. The teacher ensures that all students understand the context in order to provide a common starting point. This is particularly important in multicultural classrooms with students coming from a wide variety of ethnical backgrounds and experiences. In fact, some of the first graders are experiencing instructions in Cantonese for few months only, and not all are fluent in the instructional language. When a familiar and relevant context is used, students are more likely to demonstrate a spontaneous and meaningful approach to solve the problem and engage in class discussion, as they are connected to it.

The teacher includes a variety of questions for students to consider, in which, addition and subtraction are taught at the same time to reinforce their inverse relationship. For example, "What is the problem about? Do you think you will add or subtract?". Students are responsible for confirming/disproving each other's work. The teacher also uses gestures and a variety of tools to help students visualise and understand what is being verbalised. Vocabulary building includes constructing meanings for everyday and mathematical terminology, and making connections between the two. For instance,「共有」不等於「加」！共有：和、全部，總數是多少. Use synonyms for mathematical words such as subtract, take away, 取去：拿走了，沒有了；買了：用錢購買回來；售出：把貨品（蛋糕）賣出去；吃了：把食物（蛋糕）吃進肚子後，食物不見了，沒有了. The teacher also provides students with opportunities to practise their language skills through explaining answers for problems and composing their own problems using sentence frames (see section below). Students are also asked to "retell" someone else's strategy in Cantonese to promote participation and to communicate mathematically. By doing this, teacher can monitor how well students comprehend the problems and information being presented.



(A thank you card from a student after the lesson)

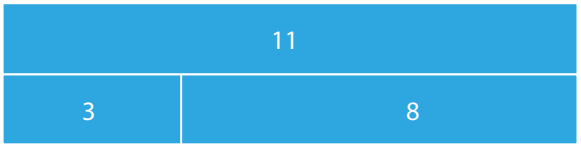
B. Combine (Part-part-whole) Situations relate to static situations involving quantities, which are considered either separately or in combination, i.e. context where everything is present the whole time, nothing changes (see Table 2). The situations involve two parts (usually) that are conceptually/mentally combined into one whole/collection. Bar model/diagram can help students model/represent the part-part-whole relationship.



There are 2 types of combine situations (adapted from Corte & Verschaffel, 1991).

Type	Example	Direction	Unknown
1	Lee has 8 momos; Lee has 5 momos; how many momos do they have altogether?	-	Superset
2	Lee and Sita have 13 momos altogether; Lee has 8 momos; how many momos does Sita have?	-	subset

C. Compare Situations involve two quantities that are compared and the difference between them. Like combine situations, they do not typically involve a physical action (see Table 3). Compare problems are harder for students to learn than either change problem or part-part-whole problems. Have the students use bar model/diagram to represent the amounts for Lee and the amounts for Sita to help make more explicit the relationship between the two quantities.



There are 6 types of compare situations (adapted from Corte & Verschaffel, 1991).

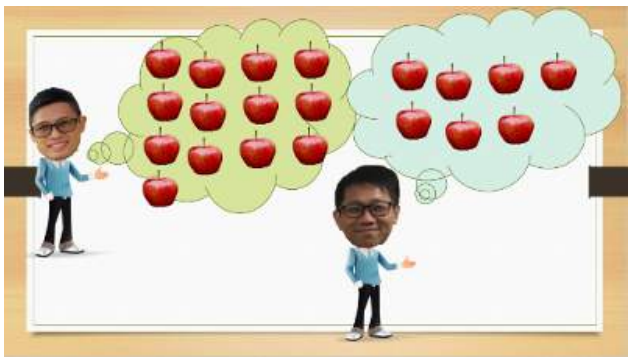
Type	Example	Direction	Unknown
1	Lee has 11 momos; Sita has 8 momos; how many momos does Lee have more than Sita?	More	Difference set
2	Lee has 11 momos; Sita has 8 momos; how many momos does Sita have less than Lee?	Less	Difference set
3	Lee has 3 momos; Sita has 8 more momos than Lee; how many momos does Sita have?	More	Compared set
4	Lee has 11 momos, Sita has 3 momos less than Lee; how many momos does Sita have?	Less	Compared set
5	Lee has 11 momos; he has 3 more momos than Sita; How many momos does Sita have?	More	Reference set
6	Lee has 8 momos; he 3 momos less than Sita; how many momos does Sita have?	Less	Reference set

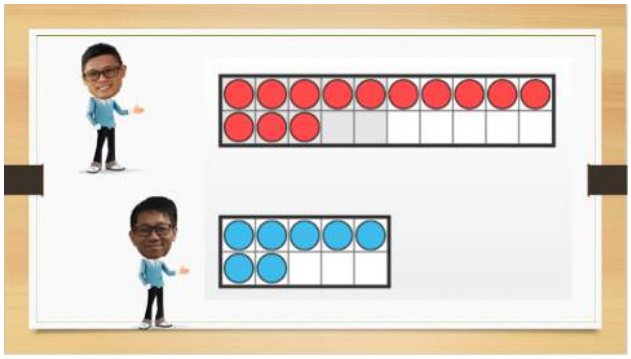
USING TOOLS TO MODEL THE SITUATIONS

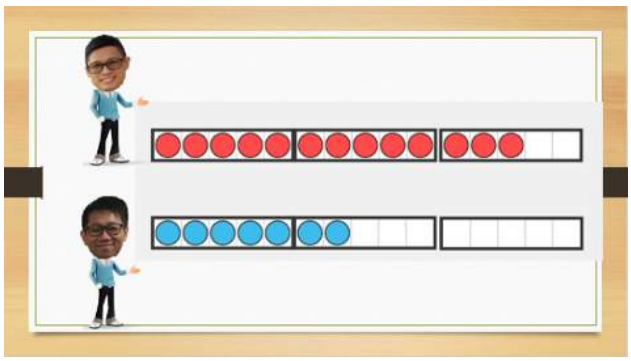
Two examples provided: One problem in which the difference is stated in terms of "how many more" and another in terms of "how much less" (or how many fewer). The language of "more", "less", "fewer" may confuse students initially, and may present a challenge in interpreting the relationship between quantities. Progress from purely pictorial to purely abstract graphical representations is shown.

Peter has thirteen apples and Steve has seven apples.

How many more apples does Peter have than Steve?

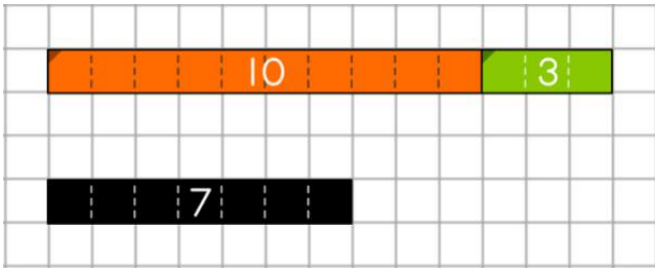




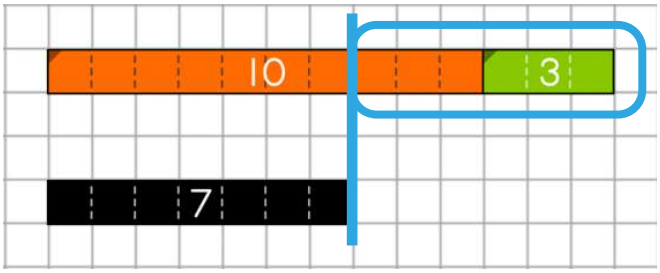


(Use 10-frame/counters as a generalised representation)

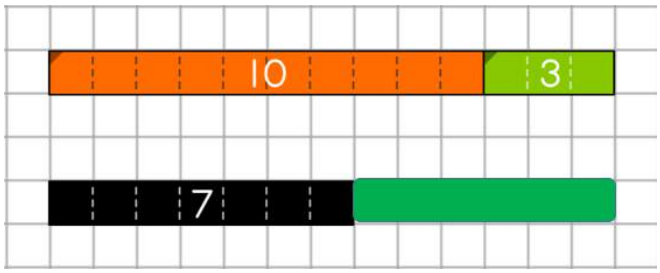
Support this comparison by using Cuisenaire rods/bar diagram to represent the two quantities. Focus on the gap between numbers, and introduce the term "difference" for describing the numerical gap between the two values.



(Use Cuisenaire rods to model the situation)



Ask: Who has more apples? How many more?



Ask: Who has fewer apples? How many fewer?

CAUTION: AVOID THE KEYWORD ONLY STRATEGY!

Van de Walle, Karp, and Bay-Williams (2019, p. 178) state that "the most important approach to solving any contextual problem is to analyse and make sense of it using all the words. The key word approach encourages children to ignore the meaning and structure of the problem". Consider the compare situations. Type 1: Lee has 11 momos; Sita has 8 momos; how many momos does Lee have more than Sita? and Type 3: Lee has 3 momos; Sita has 8 more momos than Ming; how many momos does Sita have? Both situations contain the word more, but the underlying mathematics is very different. Also, consider 樂樂吃了棒棒糖7支，思思吃了棒棒糖5支，二人共吃了棒棒糖多少支？我看「吃了」二字，應該是減法吧！ If students rely solely on the keyword strategy, they would choose the wrong operation in word problems, therefore teachers should focus on the structure of the problem as discussed earlier. Do NOT teach keywords only to solve word problems, the meaning of words in mathematics is often determined by the context.

(Vocabulary cards)

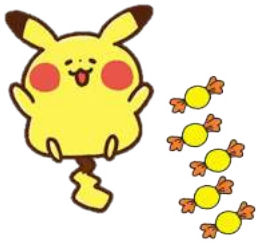
SUPPORTING CLASSROOM TALK IN MULTILINGUAL CLASSROOMS

Sentence frame is a powerful tool for students at different levels of language acquisition to practise in expressing their thinking (Dolley & Roe, 2010). When facilitating classroom talk, teachers can structure discussions in ways that provide access to emergent language learners, as well as learners with different language proficiencies.

In this lesson, the first graders are asked not just to solve problems but also to use words, pictures, and numbers to explain how they solve the contextual problems (written by the teacher) and justify their answers. Simple diagram of the problem situation can help students sort out their thinking and express their reasoning orally when solving problems. Students start with some pictures and compose problems using content words and sentence frames, which employs communication skills and develops thinking at a higher level. They are able to transit from the use of everyday language to mathematical language through the building of Chinese sentences.

加減應用題 I

根據方框中的文意，從選項中 **圈出** 適當的句子，**組成**一句**合理的應用題**：

<table>
<tr><td rowspan="3">  </td><td>**取去**2粒，</td><td rowspan="2">**共有**多少粒？</td></tr>
<tr><td>**買了**2粒，</td></tr>
<tr><td>**吃了**2粒，</td><td>**還有**多少粒？</td></tr>
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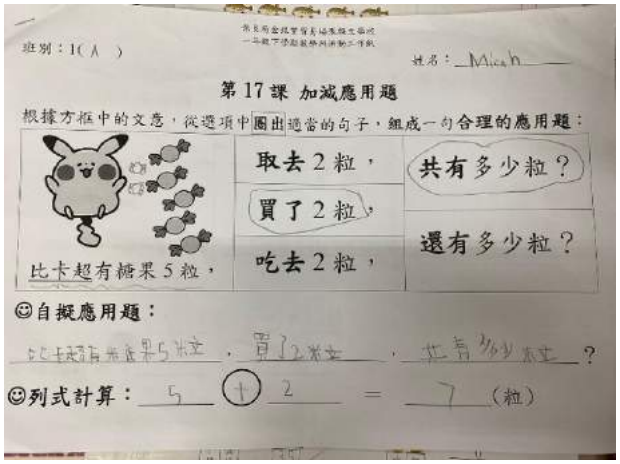
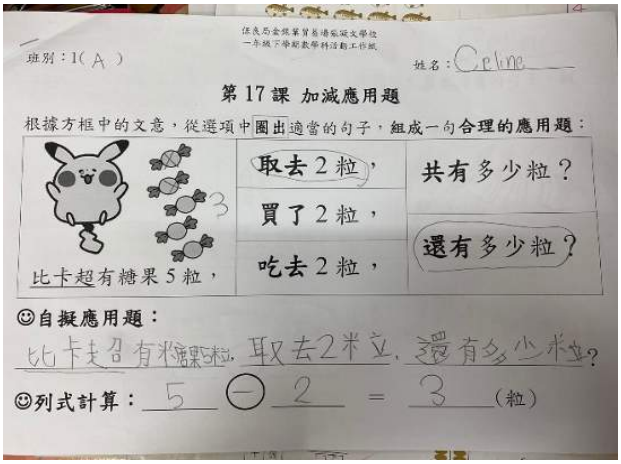
😀 **自擬應用題：**

_____，_____，_____？

😀 **列式計算：**_____ _____ ＝ _____**(粒)**





加減應用題 II

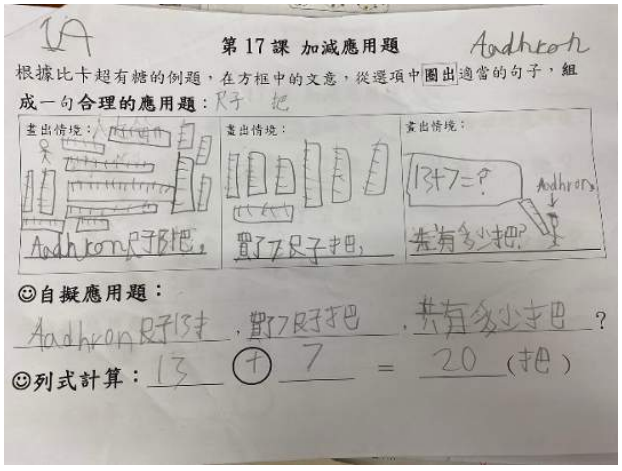
根據比卡超有糖的例題，在方框中的文意，從選項中**圈出**適當的句子，組成一句**合理的應用題**：

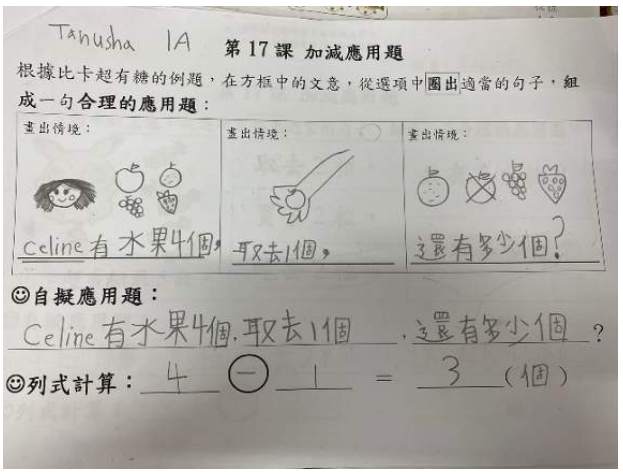
畫出情境：	**畫出情境：**	**畫出情境：**
_____	_____	_____

自擬應用題：

_____，_____，_____？

列式計算：_____ _____ ＝ _____(　　)





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