Fractions in Disguise

P4: Finding equivalent fractions and simplifying fractions

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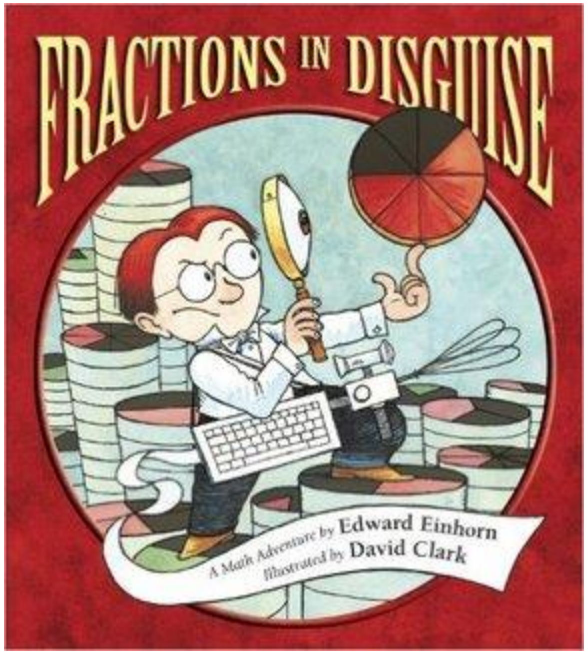
Studies have shown that if children are given the time to develop their own reasoning for at least three years without being taught standard algorithms for operations with fractions and ratios, then a dramatic increase in their reasoning abilities occurred, including their proportional thinking (Lamon, 1999 p. 5).

Finding equivalent fractions and simplifying fractions



FRACTIONS IN DISGUISE

by Edward Einhorn (2014)
Charlesbridge Publishing (USA)

George Cornelius Factor

(a.k.a. GCF, Greatest Common Factor)

https://www.youtube.com/watch?v=GhVE6FJiGOI

Polling

- How good is the book from a mathematical perspective?

  Excellent Recommended Acceptable Marginal Unacceptable

- How good is the book from a literary/language perspective?

  Excellent Recommended Acceptable Marginal Unacceptable

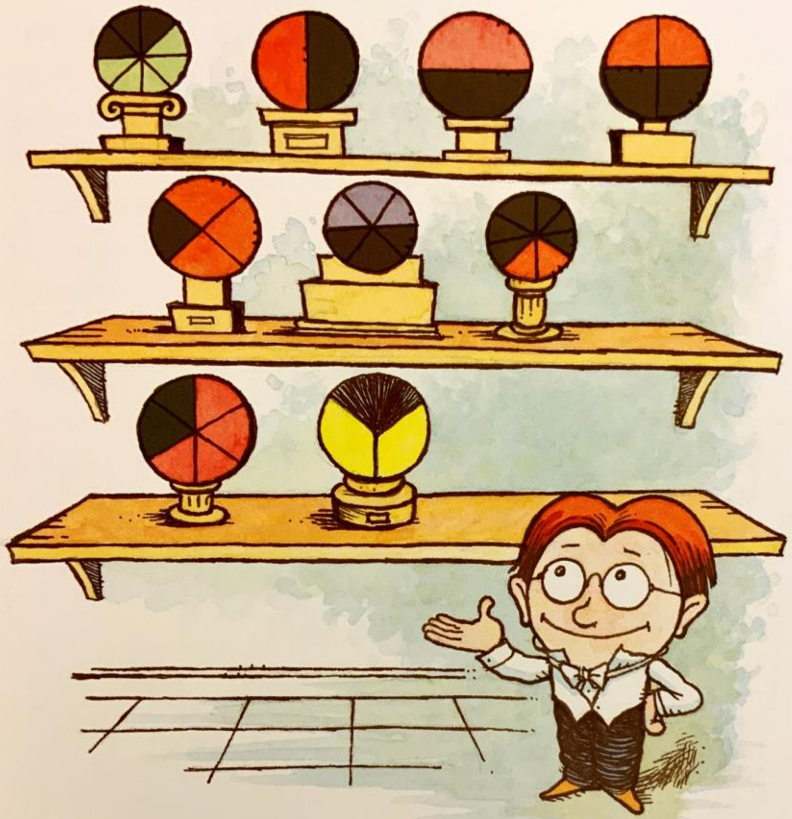
Evaluation

1. Is the book’s mathematics content (including illustrations and graphics) and language correct and accurate?
2. Is the book’s mathematics content visible and effectively presented?
3. Do the book’s mathematics content and story complement each other?
4. Does the book facilitate the student’s understanding and use of fractions?

Evaluation

1. Are the book’s illustrations and graphics text relevant, appealing, and representative of a student’s perspective?
2. Are the book’s readability and interest level developmentally appropriate for P.4?
3. Does the book respect the reader by presenting positive ethical and cultural values?

FRACTIONS IN DISGUISE



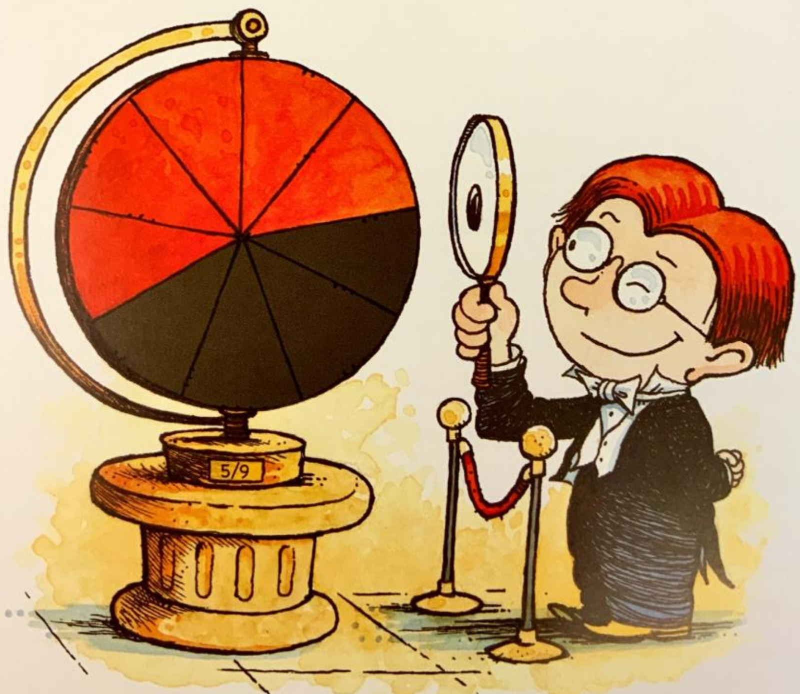
Some kids collect baseball cards. Some collect action figures. Me? I collect fractions. I've been collecting them for exactly $\frac{2}{3}$ of my life. In my bedroom, shelves full of fractions cover $\frac{3}{4}$ of the walls.

Maybe it's because I was born during a half moon. Or maybe it's because I'm $\frac{1}{4}$ genius, $\frac{1}{4}$ stubborn, $\frac{1}{3}$ determined, and $\frac{1}{6}$ crazy. But for me, it all adds up to one thing: I can't get enough of those darn fractions.

So when a brand-new $\frac{5}{9}$ went up for auction, you know I was first in line to buy it. The $\frac{5}{9}$ is a thing of beauty. When you first look at it, it looks like a $\frac{1}{2}$, but the more you look, the more you realize it's just a little bit more.

The room was filled with the regular customers: Baron von Mathematik, Madame de Géométrique, and the mysterious Dr. Brok, a former university professor rumored to have been fired for the illegal possession of a $\frac{4}{0}$.

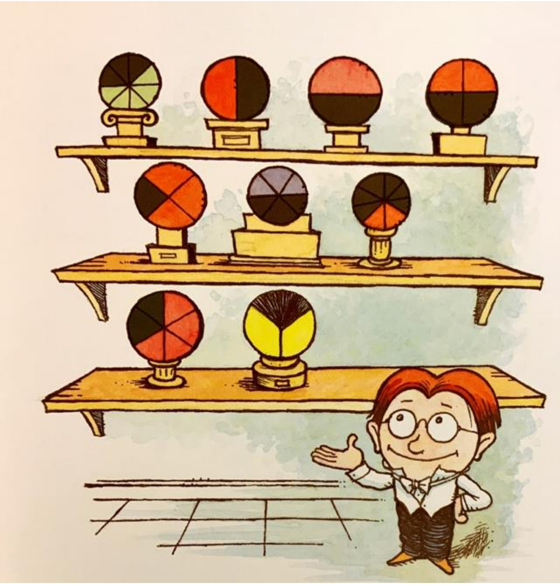
I bid $\frac{1}{2}$ of a million dollars. Madame de Géométrique bid $\frac{3}{4}$ of a million. Baron von Mathematik bid $\frac{7}{8}$ of a million. Our bids were clearly approaching one million dollars. Would we ever reach it?



PRE-READING TASKS

Prepare a mini presentation to share with us your personality and interest. 1) Describe your personality using ***fractions***. Use the following circle to help you.

Adjective	Fraction



Mr. Factor's personality:

1/4 genius, 1/4 stubborn, 1/3 determined, and 1/6 crazy

Prepare a mini presentation to share with us your personality and interest. 2) Are you a collector? What do you collect? How long have you been collection them? Where do you put them? Try to use ***fractions*** when you answer some of these questions.

4N6 Fractions (II)

Learning Objective:

3. Recognise the concepts of expanding fractions and reducing fractions

Remarks:

Students are required to recognise the concept of fractions in their lowest terms.

Textbook examples

2 Reducing fractions

1 Study the number line and carry out the group activity below.

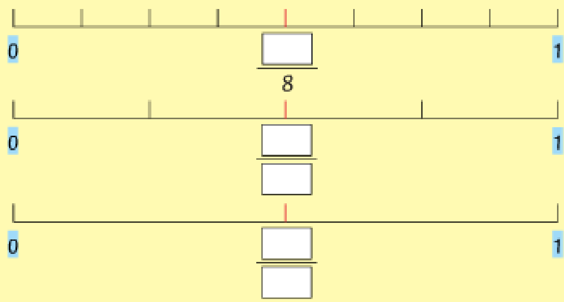
Group Activity

Fill the number line

- Work in pairs. Study the three sets of number line below. Write the correct numbers in the boxes after discussion.

My Records

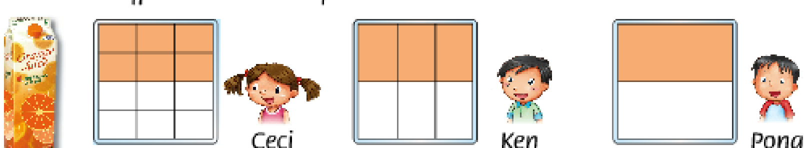
Answer



Answer

My Findings $\frac{\square}{8} = \frac{\square}{\square} = \frac{\square}{\square}$

2 Ceci, Ken and Pong separately pour fruit juice into the ice-making containers as shown below. The containers are of the same size but have different numbers of sections.



$\frac{6}{12}$ of the container filled with fruit juice

$\frac{3}{6}$ of the container filled with fruit juice

$\frac{1}{2}$ of the container filled with fruit juice

Do they use the same amount of fruit juice to make ice? Why?

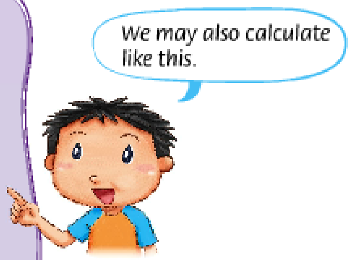
Answer

$$\frac{6}{12} = \frac{6 \div 2}{12 \div 2} = \frac{3}{6}$$

2 is a common factor of 6 and 12

$$\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$$

6 is a common factor of 6 and 12



$\because$ stands for 'Because'
$\therefore$ stands for 'Therefore'

$$\because \frac{6}{12} = \frac{3}{6} = \frac{1}{2}$$

$\therefore$ They use the same amount of fruit juice to make ice.

When dividing both the numerator and denominator by their common factors (except 1), the value of the fraction does not change. This process is called **reducing a fraction**.

Class Study

Answer ① $\frac{9}{15} = \frac{\square}{5}$ Answer ② $\frac{28}{36} = \frac{7}{\square}$ Answer ③ $\frac{13}{39} = \frac{\square}{\square}$

Higher Order Thinking Problems

Answer

Can you reduce $\frac{15}{16}$? Why?

Textbook examples

3 The simplest form of fractions

1 Let's see how to reduce a fraction.

Ceci's method:

$$\frac{6}{24} = \frac{6 \div 2}{24 \div 2}$$

2 is a common factor of 6 and 24.

$$= \frac{3}{12}$$

3 and 12 have other common factor(s) besides 1, so it can be further reduced.

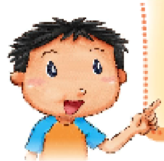
$$= \frac{3 \div 3}{12 \div 3}$$

3 is a common factor of 3 and 12.

$$= \frac{1}{4}$$

1 is the only common factor of 1 and 4, so it cannot be further reduced.

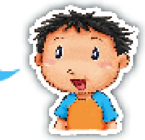
Since $\frac{1}{4}$ cannot be further reduced, $\frac{1}{4}$ is a fraction in the simplest form.



Peter's method: $\frac{\cancel{6}^{\cancel{3}^{1}}}{\cancel{24}_{\cancel{12}_{4}}} = \frac{1}{4}$

Answer I think (Ceci's / Peter's) method is simpler.

For a fraction where the numerator and denominator cannot be further reduced, the fraction is said to be in the simplest form.



Can a fraction be reduced just once to become its simplest form?

Yes. First, find out the H.C.F. of the numerator and denominator. Then, use this number to divide both the numerator and denominator separately.

2 Reduce $\frac{48}{72}$ to its simplest form.

The H.C.F. of 48 and 72 is 24.

$$\frac{48}{72} = \frac{48 \div 24}{72 \div 24}$$

Use the H.C.F. of 48 and 72 to divide both the numerator and denominator.

$$= \frac{2}{3}$$

The numerator and denominator cannot be further reduced, so the fraction is in its simplest form.

Answer **3** Reduce $\frac{45}{60}$ to its simplest form. Electronic Tools

The H.C.F. of 45 and 60 is ____.

$$\frac{45}{60} = \frac{45 \div \square}{60 \div \square}$$

$$= \frac{\square}{\square}$$

Class Study

Reduce each fraction below to its simplest form.

Answer ① $\frac{10}{18} = \square$ Answer ② $\frac{24}{96} = \square$ Answer ③ $\frac{49}{147} = \square$

Equivalent fractions

As noted by Barnett-Clarke and colleagues (2010), the words "equivalent" and "equal" mean essentially the same thing, yet students may give different meanings to them and to a third term—"same." These differences may stem from thinking of representations of physical quantities, such as $\frac{3}{4}$ and $\frac{6}{8}$ of the same thing, as "equivalent" amounts or values, but reading the equation $\frac{3}{4} = \frac{6}{8}$ aloud as "$\frac{3}{4}$ equals $\frac{6}{8}$." Moreover, students may hear the statement, "Three-fourths is the same as $\frac{6}{8}$," but they can plainly see that "$\frac{3}{4}$" does not look the same as "$\frac{6}{8}$." It is challenging to use language carefully while helping students develop an understanding of the associated meanings of the terms "equal," "equivalent," and "same."

Equivalent fractions

"EQUIVALENT" refers to the same numerical value or amount. When you talk about equivalent fractions, you try should use statements such as

½ is equivalent to 2/4

or

½ is the same amount as 2/4

and AVOID statements such as,

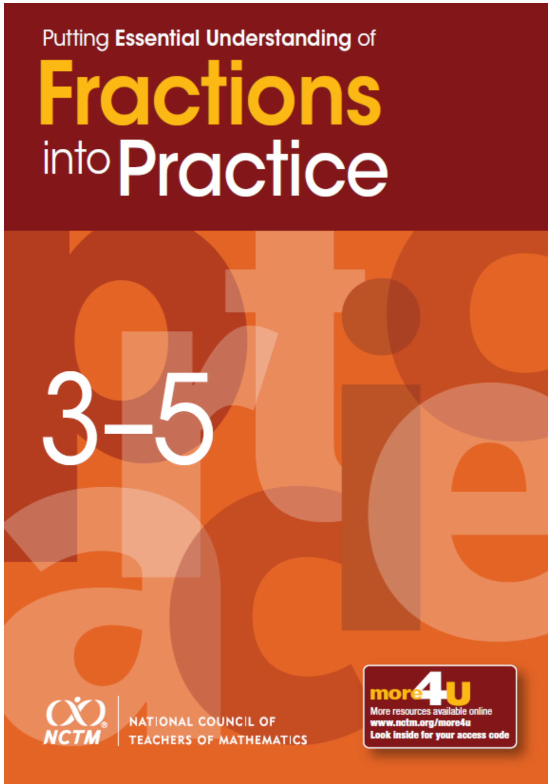
½ is the same as 2/4

or

½ looks like 2/4

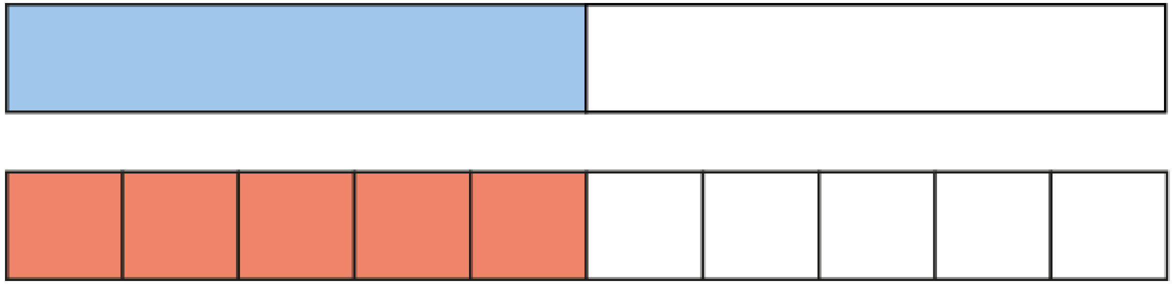
Language of fractions

What happens when a student begins with a fraction such as $\frac{8}{10}$? He can simplify this fraction to $\frac{4}{5}$ by combining each pair of tenths to make 1 fifth, thus changing the amount to $\frac{4}{5}$. Note that we use the word "simplify" rather than "reduce"—a term that can be confusing to students, suggesting that the "reduced" fraction is less than its original amount. What happens to the number of partitions that the student has? He decreases the number of partitions by half, or divides the number of partitions by 2.



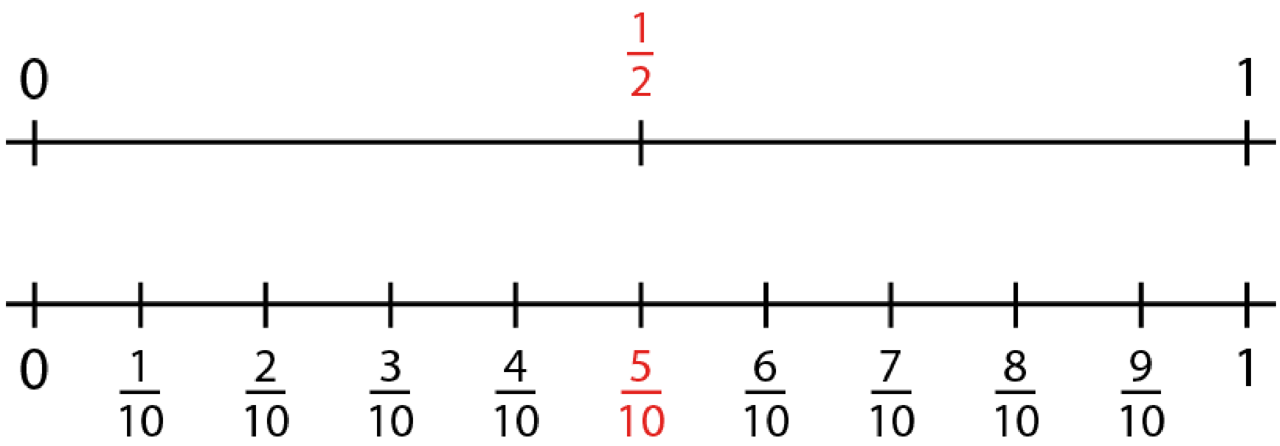
Equivalent fractions and simplifying fractions

Splitting to determine an equivalent fraction for a half



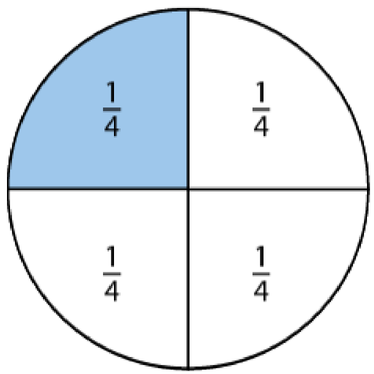
Number lines

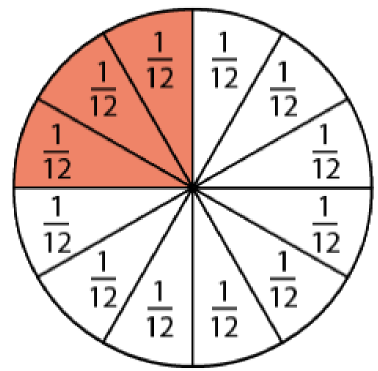
Number lines are a powerful mathematical thinking tool for solving problems and communicating ideas. And unlike area models (rectangles, circles), there is only one dimension (length) to consider.



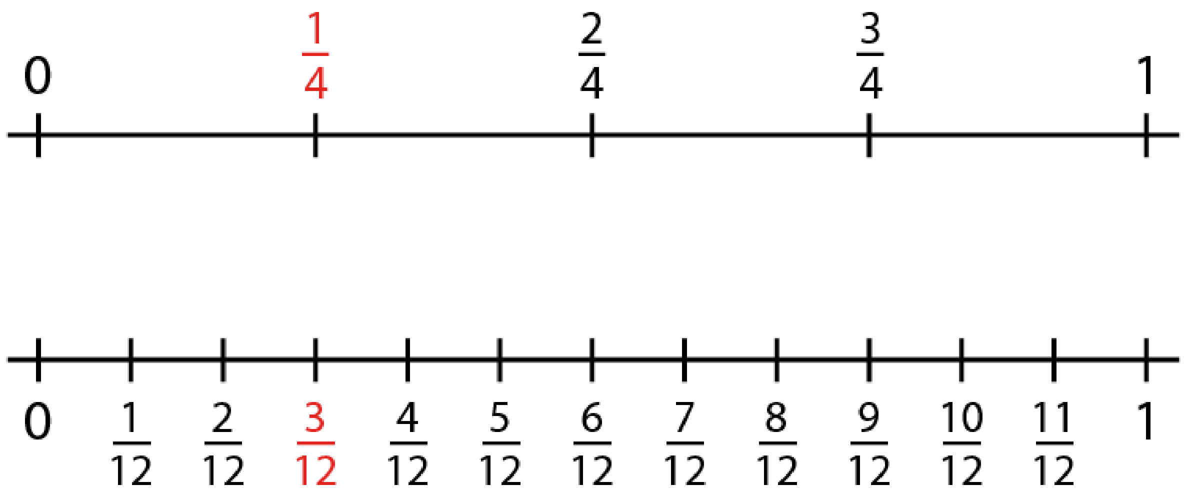
$$\frac{1}{2} = \frac{5}{10}$$

Equivalent fractions and simplifying fractions





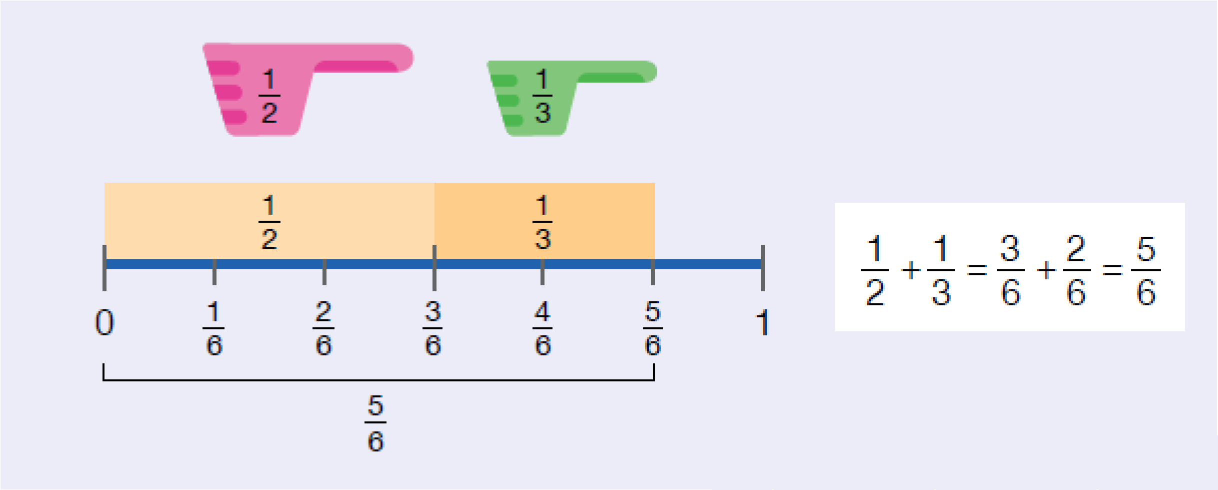
$$\frac{1}{4} \qquad \frac{3}{12}$$

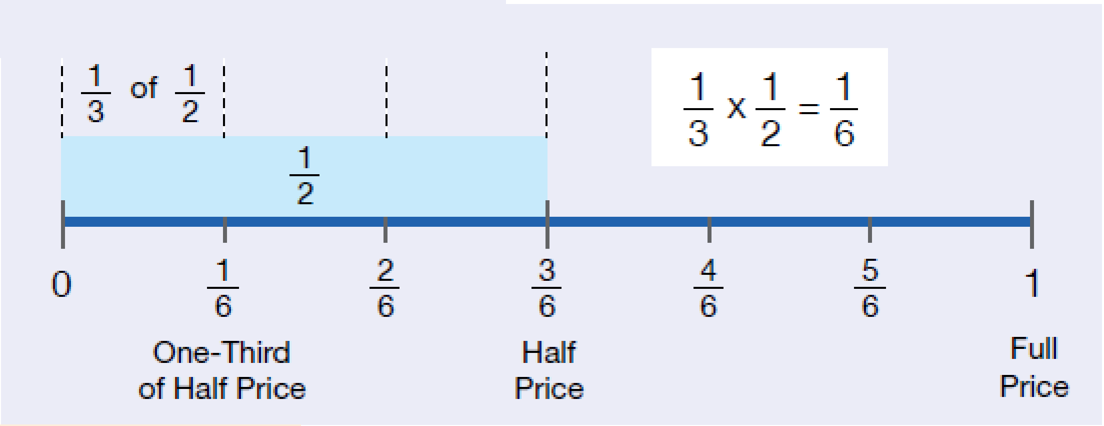


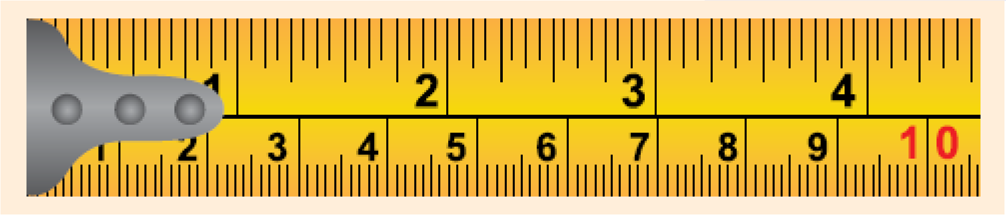
$$\frac{1}{4} = \frac{3}{12}$$

Number lines

Double number lines can show two units at once and are great to show equivalence in solving problems.



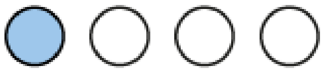




Fractions across the Curriculum (2018)

Equivalent fractions and simplifying fractions

The exploration of equivalence allows students to develop an understanding of equivalent fractions as simply being a different way of naming the same quantity.



"But how can he hope to hide it?" I asked.

"He is a master of disguise, Mr. Factor," Madame de Géométrique explained. "He can take a $\frac{1}{2}$ and turn it into a $\frac{2}{4}$ or a $\frac{3}{6}$. It's still the same fraction, but it looks different."

"So am I to understand that he could take a $\frac{3}{5}$ . . . ," I began.

". . . multiply the 3 by 4 and the 5 by 4 . . . ," continued Madame de Géométrique.

". . . and have something that looks like a $\frac{12}{20}$?" I concluded.

"But it's still $\frac{3}{5}$ really," Madame de Géométrique agreed. "He just doesn't want you to know it."

I tested it out that morning. For a long time I had owned a $^{10}/_{15}$, but I suspected that it could be another fraction in disguise.

I pointed the Reducer at it and dialed a 2. The top number (or numerator, as we call it in the trade) wavered, trying to turn into a 5, but the bottom number (or denominator) stayed the same. I dialed a 3, and the denominator tried to transform into a 5, but the numerator wouldn't budge. I dialed a 4, and nothing happened. Finally, I dialed a 5.

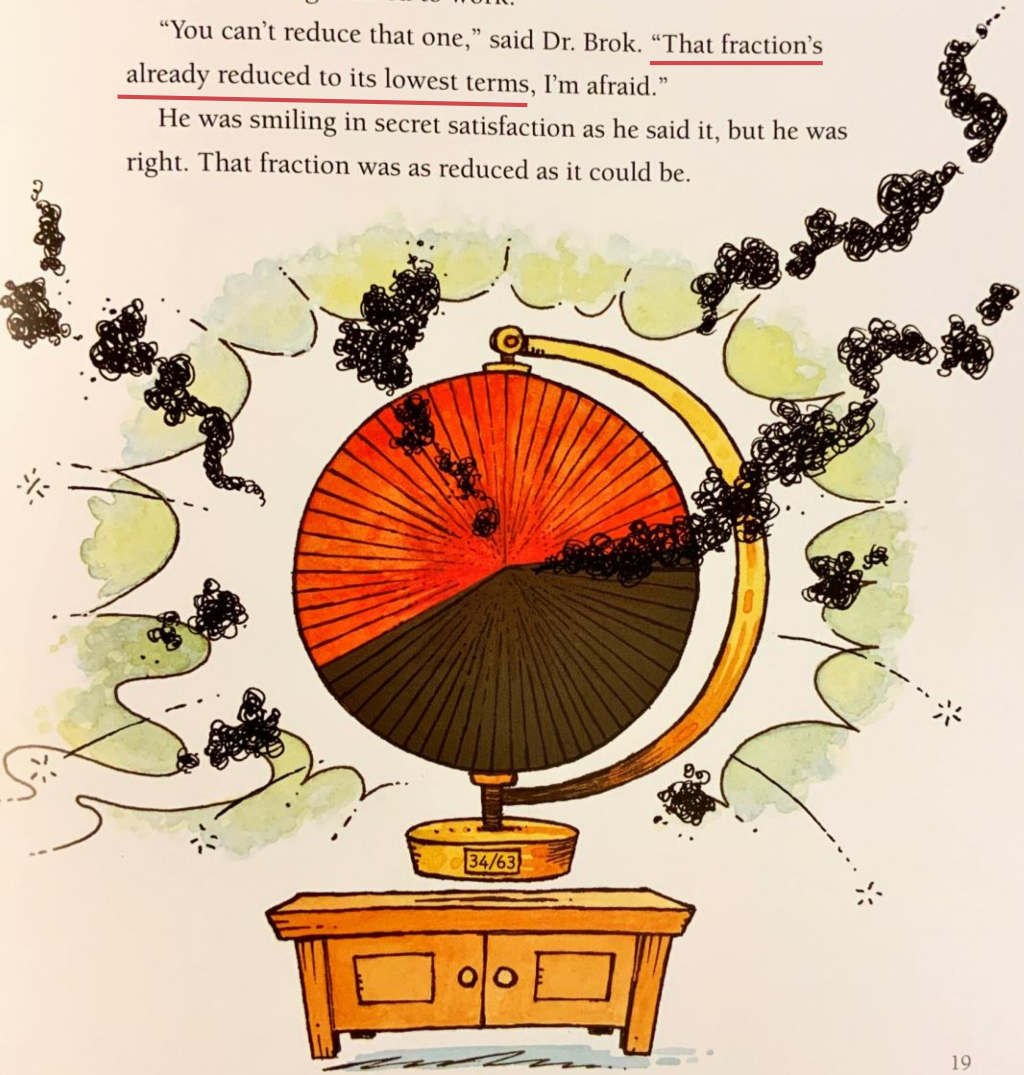
The fraction changed completely. The 10 became a 2 and the 15 became a 3, leaving me with a sleek $^{2}/_{3}$. The Reducer was ready to go.

FRACTIONS IN DISGUISE

I spotted another suspicious fraction. Could that be the $\frac{5}{9}$ in disguise? It was a $\frac{34}{63}$, and it looked familiar. I dialed a 2, 3, 4, 5, 6, 7 . . . Nothing seemed to work.

"You can't reduce that one," said Dr. Brok. "That fraction's already reduced to its lowest terms, I'm afraid."

He was smiling in secret satisfaction as he said it, but he was right. That fraction was as reduced as it could be.



Quickly I raced back to the $^{34}/_{63}$. The tiny $^{1}/_{63}$ fit perfectly into it, making the fraction a $^{35}/_{63}$. This was a fraction that could be reduced!

"Wait, no!" cried Dr. Brok.

1) Design your Reducer. List out the components using ***fractions***.

Mr. Factor's Reducer:	Your Reducer:
 *This Reducer is 1/2 ray gun and 1/2 calculator, made from a whole lot of paper clips, a whisk, some discarded computer parts, and sheer ingenuity.*	________________________________ ________________________________ ________________________________ ________________________________

2) Write your own problems about ***equivalent fractions***. Challenge your neighbour. Then check your neighbour’s answers.

1) Design your own Expander. List out the components using ***fractions*** and explain the function of your Expander.

Your Expander:	Components:
	______________________________
	______________________________
	______________________________
	______________________________
	Function:
	______________________________
	______________________________

2) Re-write the story (or part of the story) with your Expander.

TIPS

Culture is another significant element in a storytelling piece. While students re-write the story, instead of using the names from the original text such as Baron von Mathematik, Madame de Geometrique, **encourage students to play around the names of the characters with reference to their own culture.**

One cannot separate the culture from the text.
(Bharucha, 1993:70)

Brainstorm

- Where fractions can be found in the real world?
- …
- …

References

- Barnett-Clarke, C., Fisher, W., Marks, R., & Ross, S. (2010). *Developing Essential Understanding of Rational Numbers for Teaching Mathematics in Grades 3–5*. Essential Understanding Series. Reston, Va.: NCTM.
- Chval, K., Lannin, J., Jones, D., & Dougherty, B. J. (2013). *Putting Essential Understanding of Fractions into Practice in Grades 3-5*. Essential Understanding Series. Reston, Va.: NCTM.
- Einhorn, E. (2014). *Fractions in Disguise*. Charlesbridge.
- Lamon, S. J. (1999). *Teaching Fractions and Ratios for Understanding: Essential Knowledge and Instructional Strategies for Teachers*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Some of the slides are adapted from www.ncetm.org.uk/masterypd

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Teaching of Mathematics

Language of fractions

A fraction represents a number and it is read as a number, so we say “three-fourths”. This number describes a relationship between two numbers.

To aid students in developing an understanding of the meaning of the numerator as a count of the parts making up the whole and the denominator as the name for the part of the whole that is under consideration, Gunderson and Gunderson (1957) suggest that teachers initially refer to $\frac{3}{4}$ as “3-fourths” to emphasize that $\frac{3}{4}$ means 3 parts that are called “fourths.” In addition, Siebert and Gaskin (2006) recommend that teachers avoid language such as “three over four” and “three out of four” because this language can perpetuate students’ views of numerators and denominators as separate values.

Language of fractions

Knowledge of instructional strategies

As discussed on page 91, Mack (2004) posed the problem, “You have three-eighths of a medium pepperoni pizza. I give you two-eighths more of a medium pepperoni pizza. How much of a medium pepperoni pizza do you have now?” Note the strategy that she used to emphasize the unit. Rather than use numerical symbols ($\frac{3}{8}$ and $\frac{2}{8}$) in the problem, she spelled out the unit—”eighths.” Often, students encounter fractions only as numerical symbols, rather than as words. Spelling out the names can be an effective instructional strategy when it is connected with a sustained emphasis on the meaning of units—especially for English language learners.

If you want students to understand fractions better, start by focusing on unit fractions.