Multiplication
and
Division
Situations :
What the research says

by Emily S.W. Sum

When solving word problems, students "frequently choose an operation without making sense of the choice. Knowing why an operation is an appropriate choice for a solution strategy is an important part of establishing a robust understanding of mathematics" (Otto et al., 2011, p. 15). It is therefore important to provide a foundational understanding of the meaning of multiplication/division to support students' competence in problem solving and computation (Fuson, 2003). To better examine students' understanding, a **Diagnostic Assessment** can be designed to explore students' reasoning, and their prior understanding of the meaning of multiplication and division. There are evidences of students' additive and multiplicative thinking before formal instruction takes place. Examples of students' works are shown in the next section.

According to Greer (1992), the most important classes of situations involving multiplication and division of integers are: equal groups; multiplicative comparison; Cartesian product and rectangular area. Two types of division word problem can be distinguished, corresponding to division by the multiplier and division by multiplicand (see Table 1).

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Table 1: Situations Modelled by Multiplication and Division (Greer, 1992, p. 280)			
Class	**Multiplication problem**	**Division (by multiplier)**	**Division (by multiplicand)**
Equal groups	3 children each have 4 oranges. How many oranges do they have altogether?	12 oranges are shared equally among 3 children. How many does each get?	If you have 12 oranges, how many children can you give 4 oranges?
Equal measures	3 children each have 4.2 liters of orange juice. How much orange juice do that have altogether?	2.6 liters of orange juice is shared equally among 3 children. How much does each get?	If you have 12.6 liters of orange juice, to how many children can you give 4.2 liter?
Rate	A boat moves at steady speed pf 4.2 meters per second. How far does it move in 3.3 seconds?	A boat moves 13.9 meters in 3.3 seconds. What is its average speed in meters per second?	How long does it take a boat to move 13.9 meters at a speed of 4.3 metes per second?
Measure conversion	An inch is about 2.54 centimeters. About how long is 3.1 inches in centimeters?	3.1 inches is about 7.84 centimeters. About how many centimeters are there in an inch?	An inch is about 2.54 centimeters. About how long in inches is 7.84 centimeters?
Multiplicative comparison	Iron is 0.88 times as heavy as copper. If a piece of copper weights 4.3 kg how much does a piece of iron the same size weight?	Iron is 0.88 times as heavy as copper. If a piece of iron weights 3.7 kg, how much does a piece of copper the same size weight?	A college passed the top 48 out of 80 students who sat an exam. What fraction of the students passed?
Part/whole	A college passed the top 3/5 of its students in an exam. If 80 students did the exam, how many passed?	A college passed the top 3/5 of its students in an exam. If 48 passed, how many students sat the exam?	A college passed the top 48 out of 80 students who sat an exam. What fraction of the students passed?
Multiplicative change	A piece of elastic can be stretched to 3.3 times its original length. What is the length of a piece 4.2 meters long when fully stretched?	A piece of elastic can be stretched to 3.3 times its original length. When fully stretched it is 13.9 meters long. What was its original length?	A piece of elastic 4.3 meters long can be stretched to 13.9 meters. By what factor is it lengthened?
Division			
Cartesian product	If there are 3 routes from A to B, and 4 routes from B to C, how many different ways are there of going from A to C via B?	If there are 12 different routes from A to C via B and 3 routes from A to B, how many routes are there from B to C?	
Rectangular area	What is the area of a rectangle 3.3 meters long by 4.2 meters wide?	If the area of rectangle Is 13.9m ^{2} and the length is 3.3 m what is the width?	
Product of measures	If a heater use 3.3 kilowatts of electricity for 4.2 hours, how many kilowatt-hours is that?	A heater uses 3.3 kilowatts per hours. For how long can it be used on 13.9 kilowatt-hours of electricity?	

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Van de Wallen (2007) suggests that "equal groups" and "multiplicative comparison" provide a common entry point into multiplication and division in primary schools (Two division situations: **Partitive**, fair share or sharing, 24 ÷ 4, 24 divided or shared evenly, in 4 bags, and **quotitive**, subtraction or measurement, 24 ÷ 6, how many bags of 6 there are in 24).

He also emphasizes various subtypes of multiplicative situation as shown in Table 2 (extracted from Lannin, Chavl, Jones, & Dougherty, 2013, pp. 38-39).

Table 2: Problem Types Involving Multiplicative Reasoning	
Equal Groups	
Problem Types	**Examples**
Equal Groups: Result Unknown	Dusty has 4 bags of candy. Each bag has 6 pieces of candy. How many pieces of candy does Dusty have? Pat has 4 ribbons. Each ribbon is 6 inches long. How many inches of ribbon does Pat have?
Equal Groups: Size of Multiplicative Unit Unknow (Also known as *partitive division situations*)	Dusty has 24 pieces of candy. He wants to put the same number of pieces of candy. How many pieces of candy should he put into each bag? Pat has 24 inches of ribbon. She wants to use all the ribbon and cut it so there are 4 pieces that are equal in length. How long, in inches, should each piece be?
Equal Groups: Number of Multiplicative Units Unknown (Also known as *measurement division situations*)	Dusty has 24 pieces of candy. He put all of the pieces into bags with 6 pieces of candy in each bag. How many bags did Dusty use? Pat has 24 inches of ribbon. She wants to cut the ribbon to make pieces that are 6 inches long. How many pieces of ribbon can Pat make?
Comparison	
Problem Types	**Examples**
Comparison: Result Unknown	Marcia has 6 pieces of candy. Jacob has 4 times as many pieces of candy as Marcia. How many pieces of candy does Jacob have? Pat has 4 inches of ribbon. Michelle has 6 times the length of ribbon that Pat has. How long, in inches is Michelle's ribbon?
Comparison: Size of Multiplicative Unit Unknown (Also known as *partitive division situations*)	Jacob has 24 pieces of candy. He has 4 times the number of pieces of candy that Marcia has. How many pieces of candy does Marcia have? Michelle has 24 inches of ribbon. She has 6 times the length of ribbon that Pat has. How long, in inches, is Pat's ribbon?
Comparison: Number of Multiplicative Units Unknown (Also known as *measurement disivion situations*)	Jacob has 24 pieces of candy. Marcia has 6 pieces of candy. How many times as many pieces of candy does Jacob have that Marcia? Michelle has 24 inches of ribbon. Pat has 4 inches of ribbon. Michelle has how many times the length of ribbon that Pat has?

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Lannin, Chavl, Jones and Dougherty (2013) emphasise the importance to connect the situations/models with the symbolic representations for multiplication and division. "It is very important, for example, to write 4 × 5 consistently to stand for 4 groups of 5. Although 5 × 4 generates the same result as 4 × 5, many of the models for 4 × 5 differ from the models for 5 × 4. Suggesting to students that it does not matter whether they (or you) write 4 × 5 or 5 × 4 would be misleading, since they could then assume that 20 ÷ 4 is the same as 4 ÷ 20. Instead, clarifying that they are talking about 4 equal-sized groups of 5, written as 4 × 5, is important, ensuring that they can identify each group of 5 as well as the 4 groups in each phy sical representation that they create" (p. 43).

In order to help students to visualize multiplicative relationships for the problem types mentioned above, teachers should consider the specific models or representations to be used in the classroom. For example, number lines, arrays and a set of discrete objects.
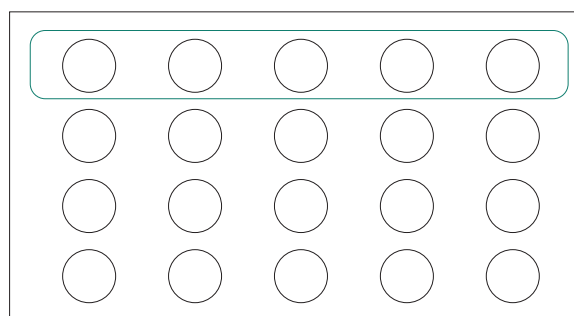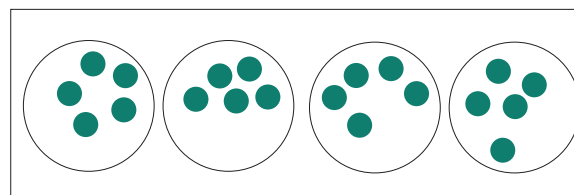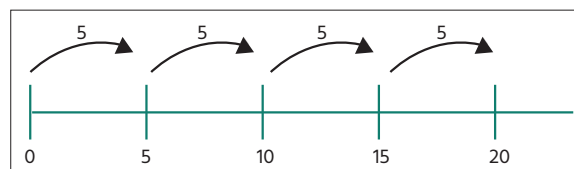
Cultural / Language Considerations

Cradall, Dale, Rhodes and Spanos (1985) suggest that if the cultural context of the world problems is unfamiliar, students may have difficulty interpreting the intent of the problem and subsequently solving it. That is, students' personal experiences may mismatch the linguistic expression used. Also, students may apply restricted experiences to mathematics problems in inappropriate ways. They may struggle with the lack of practical applications or may not relate to the situation or context presented in a problem. For instance, when solving a problem involving perimeter and area, "Jim is building a fence around a vegetable garden in his backyard···" students may not be familiar with the situation, making interpretations difficult.

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